



The Power & Use of Mathematics



Ron Larson



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Distributed by W.H. Freeman.

Printed in the U.S.A.

ISBN 13: 978-1-60840-606-7

ISBN 10: 1-60840-606-7

2 3 4 5 6 7 8 9 10

A Word from the Author



Dr. Ron Larson has written over 200 mathematics books and considers this book to be his most important. He says, "Math & YOU will liberate people from the feeling of being inept at mathematics and help them feel at ease in using math to make important decisions."

I started writing Math & YOU in the Spring of 2010. It didn't take long before the spirit and potential of the book became evident. The more I worked on the book, the more enthused I became.

All of a sudden I had a career altering realization: There is a terrible imbalance between the mathematics that is used to run our country's households, businesses, and governments, and the mathematics that we teach.

After 6th or 7th grade, the mathematics that we *teach* is almost all centered in algebra. But, whether at home, in business, or in government, the mathematics we *use* consists of little, if any, algebra!

Math & YOU changes that. In this book you will add, subtract, multiply, divide, take percents, use significant digits, and represent and interpret data. As such, you should find that much of the mathematics in this book is easy.

What is not so easy is the interpretation of the results you obtain with the mathematics.

As you read this book, remember that additional supporting material is online at math.andyou.com. I hope you love this book as much as I do.

A handwritten signature in black ink that reads "Ron Larson". The signature is written in a cursive, flowing style.

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Easy Access Social Textbook Online

The screenshot shows a web browser window with the URL <http://math.andyou.com/64>. The page title is "2.2 Markup & Discount" by Ron Larson. The page content includes a "Study Tip" box, a "Finding a Discount" section with formulas, an "EXAMPLE 3" problem about T-shirts, a "SOLUTION" section with a spreadsheet, and a "Checkpoint" section. A "Page 64" navigation bar is at the bottom.

Study Tip
Notice that for markup percent, the wholesale price is the base. However, for discount percent, the regular price is the base.

Finding a Discount
The discount on an item is the difference between the regular retail price and the discounted, or sale, price.
Discount = regular price - discounted price
To find the **discount percent**, divide the discount by the regular price.
Discount percent = $\frac{\text{discount}}{\text{regular price}}$

EXAMPLE 3 Using Discounts and Markup
You work in a clothing store. You order 25 T-shirts in assorted sizes. The wholesale price of each shirt is \$8.37. The regular retail price is \$24.99. During the next 2 months, you repeatedly mark down the price until you finally sell all 25 shirts. Find the average markup percent for the 25 shirts.

- 13 shirts at \$24.99
- 4 shirts at 25% off
- 6 shirts at 50% off
- 2 shirts at 75% off

SOLUTION
A spreadsheet works well to organize this problem.

	A	B	C	D	E	
	Wholesale Price	Regular Price	Discount Percent	Discount Price	Quantity Sold	Revenue
1						
2	\$8.37	\$24.99	0.0%	\$24.99	13	\$324.87
3	\$8.37	\$24.99	25.0%	\$18.74	4	\$74.96
4	\$8.37	\$24.99	50.0%	\$12.50	6	\$75.00
5	\$8.37	\$24.99	75.0%	\$6.25	2	\$12.50
6	Total					\$487.33

You paid $25(\$8.37) = \209.25 for the shirts. Your total markup was $487.33 - 209.25 = \$278.08$. So, your average markup percent was $\frac{\text{Total markup}}{\text{Total wholesale price}} = \frac{278.08}{209.25} = 1.329 = 132.9\%$.

Checkpoint
Find the average markup percent for the following sales record.

- 11 shirts at \$24.99
- 7 shirts at 25% off
- 3 shirts at 50% off
- 4 shirts at 75% off

Page 64 GO

View full-size printable PDFs of every page.

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- All examples and exercises are available.
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- Math Help, Consumer Suggestions, and Checkpoint Solutions
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Access a glossary of math terms, online calculators, and other useful tools.

The screenshot shows a web browser window displaying a page titled "Math Help". The page content includes:

- A search bar and navigation links for "TOC & Index" and "ABC".
- A section titled "Math Help" with a pi symbol icon.
- Text explaining technical terms like "discount" and "discount percent" used in business.
- Definitions for "Discount" and "Discount Percent".
- A list of resources: "Consumer Suggestion", "Checkpoint Solution", and "Comments (0)".

Callout boxes provide additional information:

- "Additional instruction for every lesson." points to the "Math Help" section.
- "Read tips on how to become a better consumer." points to the "Discount" and "Discount Percent" definitions.
- "View worked-out solutions for every Checkpoint." points to the "Checkpoint Solution" link.
- "Read and share comments with other students." points to the "Comments (0)" link.

In the bottom right corner, a young man is sitting on the floor, smiling, and using a laptop.

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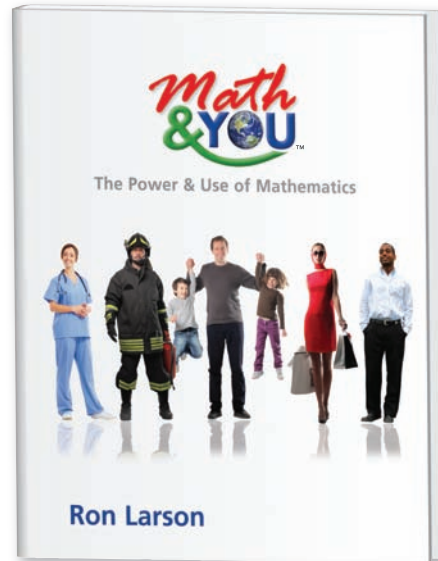
Each chapter in the book has 2 quizzes and 1 chapter review. Autograded versions are available online.

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1

The Mathematics of Calculation

1.1 Order of Operations & Formulas

- ▶ Use the order of operations to evaluate a numerical expression.
- ▶ Use a calculator to evaluate a numerical expression.
- ▶ Use the order of operations to evaluate a formula.

1.2 Rounding & Calculators

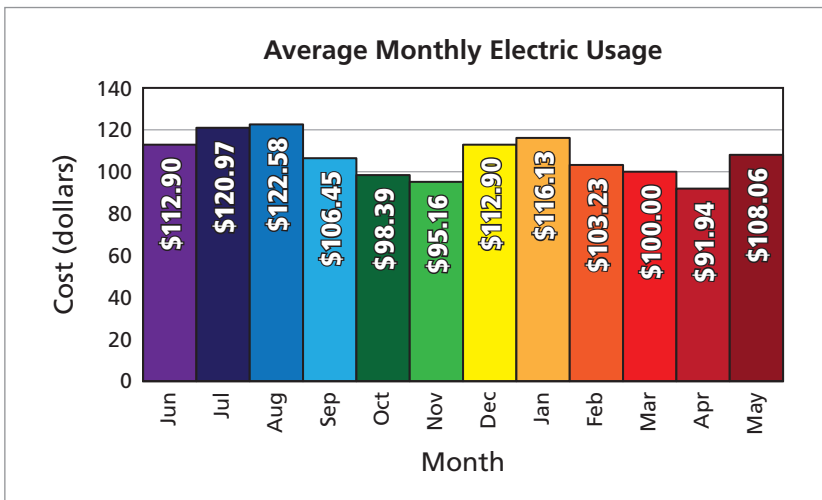
- ▶ Round numbers in a real-life context.
- ▶ Read large and small numbers.
- ▶ Understand the concept of “garbage in, garbage out.”

1.3 Using Percent

- ▶ Understand and find a percent of a number.
- ▶ Determine what percent one number is of another number.
- ▶ Use percent to represent change.

1.4 Units & Conversions

- ▶ Use unit analysis to “balance” both sides of a formula.
- ▶ Convert within a given system of measure.
- ▶ Convert between different systems of measure.



Example 1 on page 2 shows how you can calculate your reconciliation payment when you pay your electric bill on an equal-payment plan. In May, do you owe more or will you get a refund?



1.1 Order of Operations & Formulas

- ▶ Use the order of operations to evaluate a numerical expression.
- ▶ Use a calculator to evaluate a numerical expression.
- ▶ Use the order of operations to evaluate a formula.

Order of Operations

Mathematics is a language. In this language, numbers are the nouns. The operations +, −, ×, ÷, and exponentiation are the verbs. The **order of operations** is a set of rules that tells you which operations have priority.

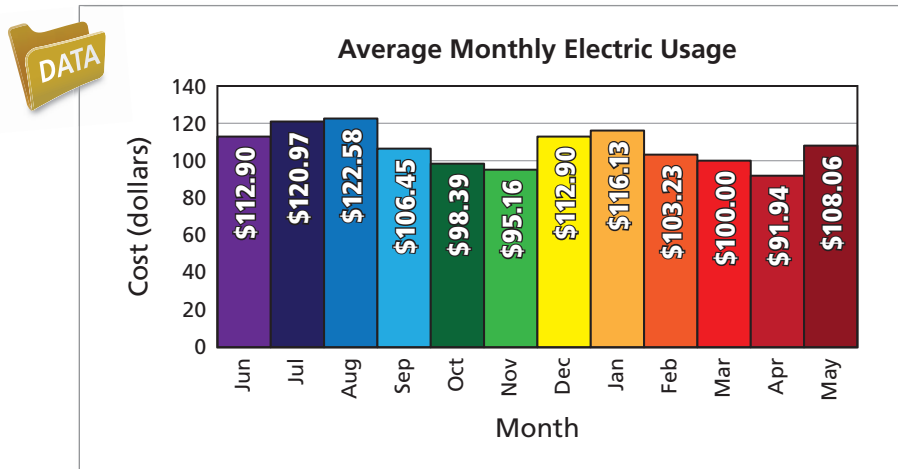
Study Tip
 A mnemonic for remembering the order of operations is "Please Excuse My Dear Aunt Sally."

Order of Operations

1. Perform operations in **P**arentheses.
2. Evaluate numbers with **E**xponents.
3. **M**ultiply or **D**ivide from left to right.
4. **A**dd or **S**ubtract from left to right.

EXAMPLE 1 Using Order of Operations

From June through April, on an equal-payment plan, you pay \$110 each month on your electric bill. How much do you owe in May?



SOLUTION

$$\begin{aligned}
 & \overbrace{(112.90 + 120.97 + 122.58 + 106.45 + 98.39 + 95.16 + 112.90)}^{\text{Amount you owe}} \\
 & \quad + 116.13 + 103.23 + 100.00 + 91.94 + 108.06 - 11(110) \quad \text{Amount you paid} \\
 & = 1288.71 - 11(110) \quad \text{Add inside parentheses.} \\
 & = 1288.71 - 1210 \quad \text{Multiply.} \\
 & = 78.71 \quad \text{Subtract.}
 \end{aligned}$$

You owe \$78.71 in May.

✓ Checkpoint

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Suppose you paid \$95 per month. How much would you owe in May?

In 2009, the greatest consumption of electricity in the United States occurred in August. The least occurred in April.

EXAMPLE 2 Using Order of Operations

There are 1950 calories in a cup of salad oil, 55 calories in an egg yolk, and 16 calories in a teaspoon of sugar. The other ingredients in the mayonnaise recipe are essentially calorie free. How many calories are in the mayonnaise recipe?

Homemade Mayonnaise	
* 2 egg yolks	* $\frac{3}{4}$ tsp salt
* $\frac{1}{2}$ tsp powdered mustard	* $\frac{1}{4}$ tsp sugar
* Pinch cayenne pepper	* $4\frac{1}{2}$ tsp white vinegar
* $1\frac{1}{2}$ cups salad oil	* 4 tsp hot water
Preparation: Beat yolks, salt, mustard, sugar, pepper, and 1 teaspoon vinegar until thick and pale yellow. Add about $\frac{1}{4}$ cup oil, drop by drop, beating vigorously. Beat in 1 teaspoon each vinegar and hot water. Add another $\frac{1}{4}$ cup oil, a few drops at a time, beating vigorously. Beat in another teaspoon each vinegar and water. Add $\frac{1}{2}$ cup oil in a fine steady stream, beating constantly. Mix in remaining vinegar and water. Slowly beat in remaining oil. Cover and refrigerate until needed. Do not keep longer than 1 week.	



Some historians say that mayonnaise originated in Bayonne, France, and was originally called bayonnaise.

SOLUTION

$$\begin{array}{l}
 \begin{array}{ccc}
 \boxed{1.5 \text{ cups}} & \boxed{2 \text{ egg yolks}} & \boxed{\text{Quarter teaspoon}} \\
 \text{of salad oil} & & \text{of sugar}
 \end{array} \\
 \downarrow \quad \quad \quad \downarrow \quad \quad \quad \downarrow \\
 \text{Total calories} = 1.5(1950) + 2(55) + 0.25(16) \\
 = 2925 + 110 + 4 \quad \quad \text{Multiply.} \\
 = 3039 \quad \quad \quad \text{Add.}
 \end{array}$$

There are 3039 calories in the recipe.

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The total number of teaspoons in the mayonnaise recipe is given by the expression below.

$$\begin{array}{ccccccccc}
 \boxed{3 \text{ tsp in an}} & \boxed{\text{Mustard}} & \boxed{48 \text{ tsp}} & \boxed{\text{Salt}} & \boxed{\text{Sugar}} & \boxed{\text{Vinegar}} & \boxed{\text{Water}} \\
 \text{egg yolk} & & \text{in a cup} & & & & \\
 \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
 2(3) + 0.5 + 1.5(48) + 0.75 + 0.25 + 4.5 + 4
 \end{array}$$

- How many teaspoons are in the recipe?
- How many calories are in a teaspoon of mayonnaise?

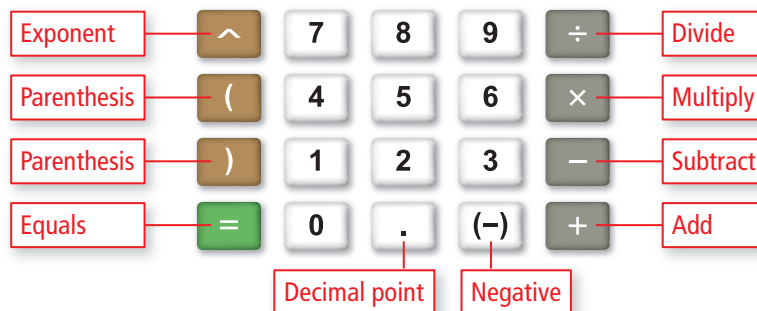
Calculators and Order of Operations

Some calculators use the standard order of operations and some do not. Rather than relying on your calculator to perform the operations in the correct order, you should use a calculator that has parentheses. Also, for this book, you will need a calculator with an exponent key.

Study Tip
 Here is an example of the exponent key. There are 12^3 cubic inches in 1 cubic foot.

1 2 ^ 3 =

Recommended Calculator Keys



EXAMPLE 3 Using a Calculator

You are taking a 3-credit evening course. The cost is \$150 for registration and \$219 for each credit. Which of the following keystroke sequences is better for finding the total cost? Explain your reasoning.



- a. 1 5 0 + (3 × 2 1 9) =
- b. 1 5 0 + 3 × 2 1 9 =

SOLUTION

- a. This sequence is better. You are forcing the calculator to multiply 3 by 219 before adding 150.

$$\begin{aligned} \text{Total cost} &= 150 + 3(219) \\ &= 150 + 657 && \text{Multiply.} \\ &= 807 && \text{Add.} \end{aligned}$$

The correct total cost is \$807.

- b. On some calculators, this keystroke sequence gives an incorrect total because it adds 150 and 3 to get 153 and then multiplies by 219 to get a total of \$33,507, which is an unreasonable answer.

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Your cell phone bills for 3 months are \$50, \$62, and \$73. Which of the following keystroke sequences is better for finding your average monthly bill? Explain your reasoning.

- c. 5 0 + 6 2 + 7 3 ÷ 3 =
- d. (5 0 + 6 2 + 7 3) ÷ 3 =

Using and Evaluating Formulas

A **formula** is an equation that relates one quantity to one or more other quantities. For instance, the area of a rectangle is given by the formula

$$\text{Area} = (\text{base})(\text{height}) \quad \text{or} \quad A = bh.$$

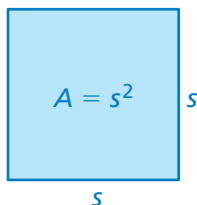
Study Tip

Another formula for the area of a rectangle is “length times width.”

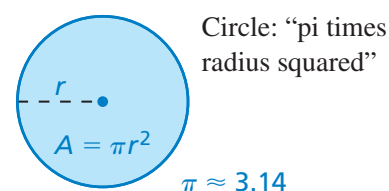
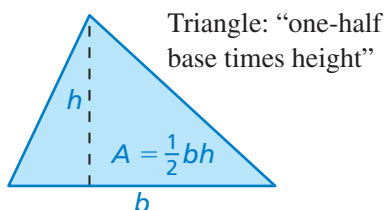
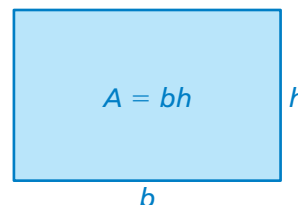
$$A = \ell w$$

Common Area Formulas

Square: “side squared”



Rectangle: “base times height”



EXAMPLE 4 Buying Floor Tiles

12 in.



12 in.

Ceramic tile has been around for at least 4000 years. Tiled surfaces have been found in the ruins of Egypt, Babylon, and Greece.

You are buying tiles for your kitchen floor. The room is 8 feet by 12 feet. The tiles are 12-inch squares that cost \$8.97 each. The tiles come in boxes of 15. You should order an extra 20 tiles to allow for waste.

- How many boxes should you order?
- What is the total cost of your order?

SOLUTION

- The area of the room is 8×12 , or 96 square feet. So, you need 96 tiles plus an extra 20 tiles for waste. This is a total of 116 tiles. Because there are 15 tiles in a box, you should order 8 boxes, giving you 120 tiles.
- The total cost of your order is

$$\begin{array}{cc} \boxed{\text{Number of tiles}} & \boxed{\text{Cost per tile}} \\ \downarrow & \downarrow \\ 120 \times 8.97 & = \$1076.40. \end{array}$$

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You are buying tiles for 2 bathroom floors. One room is 8.5 square feet. The other room is 9 feet by 10 feet. The tiles are 12-inch squares that cost \$7.65 each. The tiles come in boxes of 12. You should order an extra 20 tiles to allow for waste.

- How many boxes should you order?
- What is the total cost of your order?

Formulas for Distance, Rate, and Time

Distance = d

$$d = rt$$

Distance equals
rate times time.

Rate = r

$$r = \frac{d}{t}$$

Rate equals distance
divided by time.

Time = t

$$t = \frac{d}{r}$$

Time equals distance
divided by rate.

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You can access a distance, rate, and time calculator at *Math.andYou.com*.

EXAMPLE 5 Using the Distance Formula

Each morning, you run 7.5 miles in 45 minutes. You weigh 155 pounds. How many calories do you burn running each morning?

Calories Burned During 1 Hour of Activity

Activity	Weight		
	130 lb	155 lb	190 lb
Bicycling, 12 mph	472	563	690
Billiards	148	176	216
Fishing from boat	148	176	216
Golfing	236	281	345
Running, 10 mph	944	1126	1380
Running, 8 mph	797	950	1165
Swimming laps	590	704	863
Volleyball at beach	472	563	690



According to the Centers for Disease Control and Prevention, people who exercise regularly lower their risk of heart disease, stroke, and colon cancer.

SOLUTION

You are given the time and the distance. Your rate is given by

$$\begin{aligned} r &= \frac{d}{t} \\ &= \frac{7.5 \text{ miles}}{0.75 \text{ hour}} && \text{Write 45 minutes as 0.75 hour.} \\ &= 10 \frac{\text{miles}}{\text{hour}} \end{aligned}$$

Because 45 minutes is $\frac{3}{4}$ of an hour, you burn $\frac{3}{4}$ of 1126 calories.

$$\text{Calories burned} = 0.75(1126) = 844.5$$

So, you burn about 850 calories each morning.

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For lunch, you eat a third-pound hamburger with 590 calories, a medium cola with 210 calories, and a large order of French fries with 510 calories. How long do you have to run at 10 miles per hour to burn the calories you eat at lunch?

Study Tip

Notice the similarity between these formulas and the ones for distance, rate, and time.

Formulas for Earnings, Rate, and Time

Earnings = E

$E = rt$

Earnings equal
rate times time.

Rate = r

$r = \frac{E}{t}$

Rate equals earnings
divided by time.

Time = t

$t = \frac{E}{r}$

Time equals earnings
divided by rate.

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You can access an earnings, rate, and time calculator at *Math.andYou.com*.

EXAMPLE 6 Comparing Job Offers

Which job offer has the better total compensation? Explain your reasoning.

a.

Salary Rate:	\$30 per hour
401(k):	5% matching
Health Insurance:	\$600 per month

b.

Salary Rate:	\$59,000 per year
401(k):	6% matching
Health Insurance:	\$900 per month
Profit Sharing:	\$0–\$20,000 per year

SOLUTION

a. There are 52 weeks in a year. At 40 hours a week, you work 2080 hours in a year. Your yearly earnings are

$$E = \left(30 \frac{\$}{\text{hr}}\right)(2080 \text{ hr}) = \$62,400.$$

5% of this is \$3120 (see Section 1.3). So, your total compensation is

$$62,400 + 3120 + 12(600) = \$72,720.$$

b. 6% of \$59,000 is \$3540 (see Section 1.3). Your total compensation is

$$59,000 + 3540 + 12(900) = \$73,340.$$

So, even without profit sharing, this total compensation is better. If the company has a profitable year, the total compensation could be *much* better.

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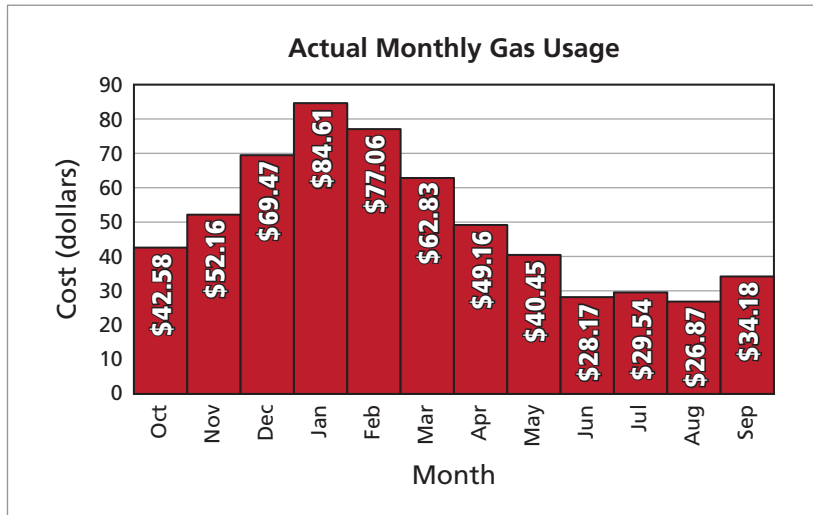
c. You get a third job offer. How does it compare with the other two offers?

Salary Rate:	\$4800 per month
401(k):	4% matching
Health Insurance:	\$1200 per month
Sales Commission:	\$0–\$3000 per month

1.1 Exercises

DATA **Equal-Payment Plan** In Exercises 1 and 2, use the graph. (See Example 1.)

- From October through August, on an equal-payment plan, you pay \$50 each month on your gas bill. How much do you owe in September?
- You are on an equal-payment plan. What is the least amount that you can pay each month from October through August to owe nothing in September?



DATA **Equal-Payment Plan** In Exercises 3 and 4, use the table. (See Example 1.)

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
\$84.02	\$86.59	\$77.65	\$72.10	\$68.26	\$62.06	\$63.47	\$65.19	\$60.12	\$67.61	\$75.36	\$81.98

- From January through November, on an equal-payment plan, you pay \$75 each month on your electric bill. How much do you owe in December?
- You are on an equal-payment plan. What is the least amount that you can pay each month from January through November to owe nothing in December?

Old-Fashioned Pink Lemonade
* 1 1/2 cups white sugar
* 6 3/4 cups water
* 1 1/2 cups fresh lemon juice
* 3/4 cup cranberry juice

- Pink Lemonade** There are 774 calories in a cup of sugar, 65 calories in a cup of fresh lemon juice, and 137 calories in a cup of cranberry juice. (See Example 2.)

 - How many calories are in the old-fashioned pink lemonade recipe?
 - This recipe makes nine servings. How many calories are in one serving?

- Fudge** There are 99 calories in an ounce of cream cheese, 12 calories in a teaspoon of vanilla extract, 389 calories in a cup of powdered sugar, and 145 calories in a square of chocolate. Salt has 0 calories. (See Example 2.)

 - How many calories are in the fudge recipe?
 - This recipe uses an 8-inch by 8-inch dish. How many calories are in a 1-inch by 1-inch piece of fudge?

Homemade Fudge
* 6 oz cream cheese, softened
* 1/8 tsp salt
* 1/2 tsp vanilla extract
* 4 cups powdered sugar, sifted
* 4 squares unsweetened chocolate

7. **Parking** You buy a campus parking permit for \$250. During the year, you get four \$15 parking tickets. Which of the following keystroke sequences is better for finding your total parking cost? Explain your reasoning. (See Example 3.)

a. $(2 5 0 + 4) \times 1 5 =$

b. $2 5 0 + (4 \times 1 5) =$

8. **Books** Your books for this semester cost \$95, \$120, and \$115. Which of the following keystroke sequences is better for finding your average book cost? Explain your reasoning. (See Example 3.)

a. $(9 5 + 1 2 0 + 1 1 5) \div 3 =$

b. $(9 5 + 1 2 0) + 1 1 5 \div 3 =$

9. **Pizza Party** You buy 3 pizzas for \$10 each and 5 bottles of cola for \$2 each. Which of the following keystroke sequences is better for finding the total cost? Explain your reasoning. (See Example 3.)

a. $(3 \times 1 0) + (5 \times 2) =$

b. $(3 \times 1 0) + 5 \times 2 =$

10. **Depreciation** A new car worth \$25,000 depreciates at a rate of 50% every 3 years. The value of the car after 6 years is $25,000(0.5^2)$. Which of the following keystroke sequences is better for finding the value of the car? Explain your reasoning. (See Example 3.)

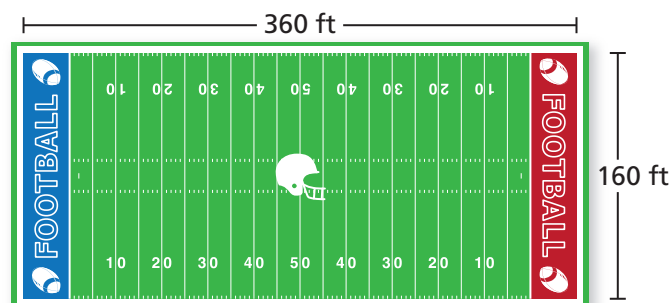
a. $(2 5 0 0 0 \times . 5) ^ 2 =$

b. $2 5 0 0 0 \times (. 5 ^ 2) =$

Football Field In Exercises 11 and 12, use the diagram of the football field. (See Example 4.)

11. You are buying lawn seed for the football field. A 10-pound bag of lawn seed covers 2000 square feet.

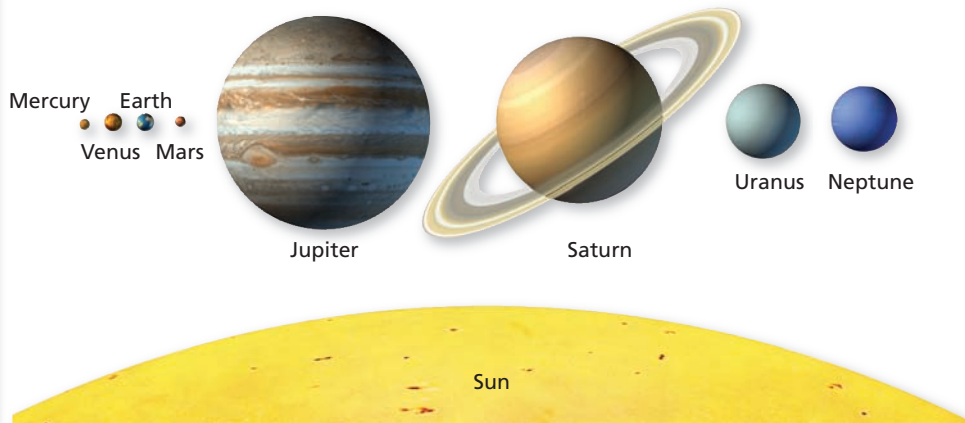
- How many 10-pound bags of lawn seed should you buy?
- A 10-pound bag of lawn seed costs \$27.79. What is the total cost of your purchase?



12. Latex paint is used to paint the end zones of the football field. Each gallon of paint must be diluted with 1 gallon of water. One gallon of diluted latex paint covers 100 square feet.
- Each end zone is 30 feet long. How many gallons of undiluted latex paint are needed to paint both end zones?
 - Why do you think the paint is diluted?

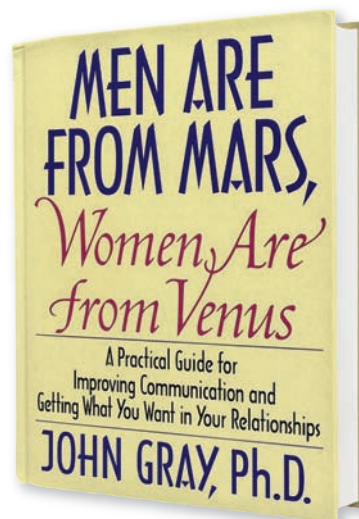
Solar System In Exercises 13–16, use the table. The speed of light is about 299,792 kilometers per second. (See Examples 5 and 6.)

Planet	Mean Distance from the Sun (km)
Mercury	57,909,175
Venus	108,208,930
Earth	149,597,890
Mars	227,936,640
Jupiter	778,412,020
Saturn	1,426,725,400
Uranus	2,870,972,200
Neptune	4,498,252,900



Saturn is the only planet in the Solar System that is less dense than water. It would float in a bathtub if you could build a bathtub big enough.

- How long does it take sunlight to reach Earth?
- How long does it take sunlight to reach Neptune?
- In the book *Men Are from Mars, Women Are from Venus*, the author John Gray suggests that communication is a key component to successful relationships among couples. Data transmission through space occurs at the speed of light. How long would it take for a text message sent from Venus to reach Mars?
- A meteor streaks through Jupiter’s atmosphere, causing a bright flash. How long before the flash is seen from Earth?



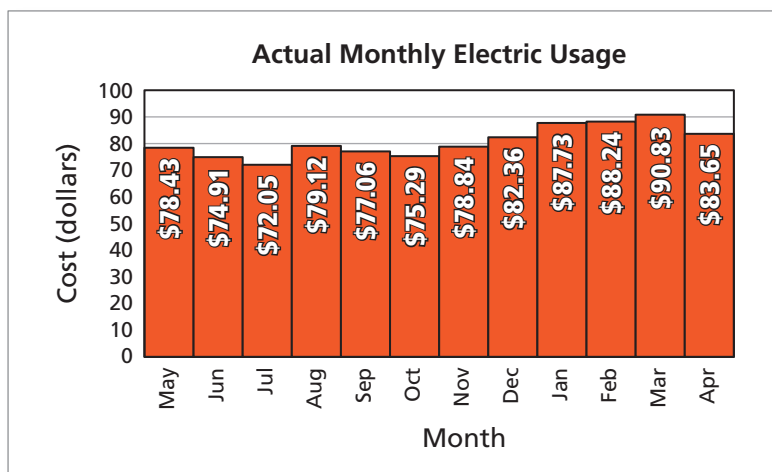
- Pluto** From its discovery in 1930 until 2006, Pluto was considered the Solar System’s ninth planet. In 2006, Pluto was reclassified as a dwarf planet because of its low mass and its similarity to other dwarf planets beyond Neptune in the Kuiper Belt. To better understand Pluto, NASA planned a mission to Pluto called New Horizons. The spacecraft launched on January 19, 2006, and is supposed to reach Pluto 9.5 years (83,220 hours) later after traveling about 3 billion miles. What is the average speed of the spacecraft in miles per hour? (See Example 5.)
- Light-Year** What is a light-year? How is the term *light-year* used?

▶ Extending Concepts



19. Equal-Payment Plan From May through March, on an equal-payment plan, you pay \$90 each month on your electric bill.

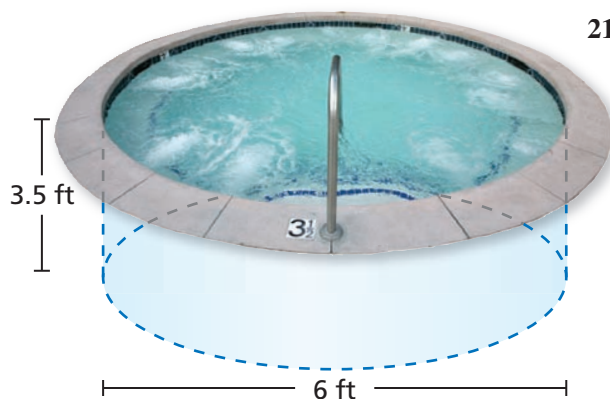
- How much do you owe in April?
- Interpret your answer to part (a).
- What should you pay each month to have 12 equal monthly payments?



20. Ink Cartridge A combo pack has 2^3 ink cartridges. A shipment has 6^2 combo packs. Which of the following keystroke sequences is better for finding the total number of ink cartridges in a shipment? Explain your reasoning.

a.

b.



21. Hot Tub The inside of a circular hot tub is lined with 1-inch by 1-inch tiles.

- Explain how to estimate the number of tiles needed for the hot tub.
- The hot tub is 3.5 feet deep and has a diameter of 6 feet. What is the minimum number of tiles needed? (The formula for the circumference of a circle is $C = \pi d$.)

Job Offers In Exercises 22–24, use the job offers shown.

Offer A

Salary Rate:	\$3200 per month
401(k):	3% matching
Health Insurance:	\$850 per month
Sales Commission:	\$0–2500 per month

Offer B

Salary Rate:	\$25 per hour
401(k):	5% matching
Health Insurance:	\$650 per month
Profit Sharing:	\$0–15,000 per year

- What are the minimum and maximum yearly total compensations for each job offer?
- Which job offer do you think is better? Explain your reasoning.
- You decide to make a counteroffer to the company you did not choose in Exercise 23. How does your counteroffer differ from the original? Explain your reasoning.

1.2 Rounding & Calculators

- ▶ Round numbers in a real-life context.
- ▶ Read large and small numbers.
- ▶ Understand the concept of “garbage in, garbage out.”



Rounding Numbers in a Real-Life Context

People tend to think that mathematics gives exact answers. In a real-life context, however, listing “exact” calculator displays is often unreasonable. It is usually reasonable to **round** the calculator display according to the context of the real-life application.

Rounding Numbers

Determine your rounding digit and look at the digit to the right of it.

1. **Rounding Down:** If the digit is 0, 1, 2, 3, or 4, do not change the rounding digit. All digits to the right of the rounding digit become 0.
2. **Rounding Up:** If the digit is 5, 6, 7, 8, or 9, add 1 to the rounding digit. All digits to the right of the rounding digit become 0.

EXAMPLE 1 Rounding Numbers in Context

Round the defense spending per person for each country so it is reasonable for the context.

a.
$$\frac{\text{2010 U.S. Defense Department budget}}{\text{2010 population of United States}} = \frac{\$660 \text{ billion}}{310 \text{ million}} \approx \$2129.03226 \text{ per person}$$

b.
$$\frac{\text{2010 China's Defense Department budget}}{\text{2010 population of China}} = \frac{\$100 \text{ billion}}{1.340 \text{ billion}} \approx \$74.62687 \text{ per person}$$

SOLUTION

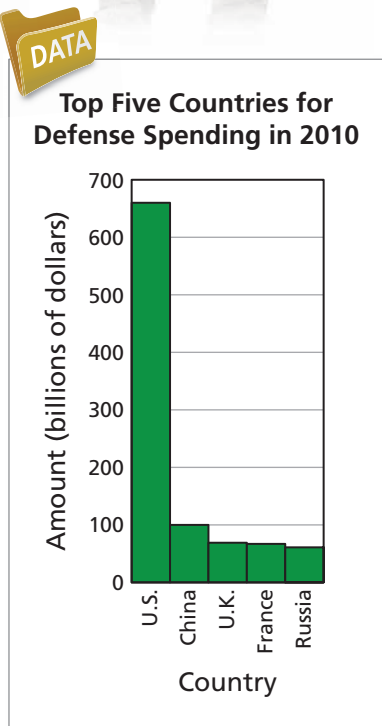
None of the numbers given in the problem are exact. So, rounding is reasonable.

- a. A reasonable answer is that defense spending in the United States is about \$2130 per person.
- b. A reasonable answer is that defense spending in China is about \$75 per person.

✓ Checkpoint

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In 2010, the population of the United Kingdom was about 62 million. Estimate its defense spending per person.



EXAMPLE 2 Comparing Annual Fuel Cost

Assume that you drive 15,000 miles in a year. Use the information given to estimate the annual fuel cost based on a fuel price of \$2.68 per gallon for 87 octane and \$2.92 per gallon for 93 octane.



	a. 2011 Toyota Prius	b. 2011 Infiniti EX35
Miles on a Tank*	536 miles	360 miles
Tank Size	11.9 gal	20.0 gal
Recommended Octane	87	93

*Based on 45% highway and 55% city driving.

SOLUTION

- a. This car can travel 536 miles on a tank of gas. To drive 15,000 miles, you need the following number of tanks.

$$\frac{15,000 \text{ miles}}{536 \text{ miles per tank}} \approx 27.985 \text{ tanks} \quad \text{See page 34.}$$

Each tank contains 11.9 gallons, so the number of gallons you use is

$$(27.985 \text{ tanks}) \left(11.9 \frac{\text{gallons}}{\text{tank}} \right) = 333.0215 \text{ gallons.}$$

At \$2.68 per gallon, your annual fuel cost is

$$(333.0215 \text{ gallons}) \left(2.68 \frac{\text{dollars}}{\text{gallon}} \right) = \$892.49762.$$

Your annual fuel cost is about \$900. In the context of the question, it is misleading to specify an annual cost of \$892.50. This would lead your reader to believe that you know more about the context than you do.

- b. This car can travel 360 miles on a tank of gas. To drive 15,000 miles, you need the following number of tanks.

$$\frac{15,000 \text{ miles}}{360 \text{ miles per tank}} \approx 41.667 \text{ tanks} \quad \text{See page 34.}$$

Each tank contains 20 gallons, so the number of gallons you use is

$$(41.667 \text{ tanks}) \left(20 \frac{\text{gallons}}{\text{tank}} \right) = 833.34 \text{ gallons.}$$

At \$2.92 per gallon, your annual fuel cost is

$$(833.34 \text{ gallons}) \left(2.92 \frac{\text{dollars}}{\text{gallon}} \right) = \$2433.3528.$$

Your annual fuel cost is about \$2400 or \$2500.

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How many miles per gallon does each car get?

- c. A 2011 Toyota Prius d. A 2011 Infiniti EX35

Study Tip

Notice that listing the units, such as gallons per tank, helps determine the units of the answer.

The United States uses about 140 billion gallons of gasoline in a year. Many countries place high taxes on gasoline. Some people have suggested a tax of at least \$1 per gallon in the United States.

Study Tip

The integer b tells you how many places to move the decimal point in the number a .

Positive exponent:

Move right

$$5.1 \times 10^8 = \underbrace{510000000.}_{\text{move 8 places right}}$$

Negative exponent:

Move left

$$2.4 \times 10^{-8} = \underbrace{0.000000024}_{\text{move 8 places left}}$$

Reading Large and Small Numbers

When numbers are too large or too small to be conveniently written in standard decimal notation, most calculators switch to *scientific* or *exponential notation*.

Exponential Notation

In **exponential notation**, numbers are written as a times a power of 10,

$$a \times 10^b$$

where a is at least 1 and less than 10, and b is an integer. Here are two examples.

Standard Decimal Notation

$$6,830,000,000$$

$$0.0000000000683$$

Exponential Notation

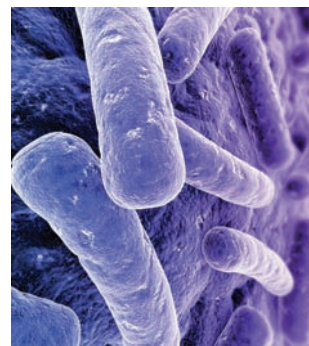
$$6.83 \times 10^9$$

$$6.83 \times 10^{-11}$$

EXAMPLE 3 Describing Large and Small Numbers

Describe the numbers in the article about bacteria.

It is estimated that 500 to 1000 species of bacteria live in the human digestive system, and a roughly similar number live on the skin. Bacteria cells are much smaller than human cells (typically 3×10^{-6} meter in length), and there are at least 10 times as many bacteria as human cells in the body (approximately 10^{14} versus 10^{13}). There are approximately 5×10^{30} bacteria on Earth.

**SOLUTION**

Length of a bacteria cell:

$$3 \times 10^{-6} \text{ meter} = 0.000003 \text{ meter} \quad \text{3 millionths}$$

Number of bacteria in a human:

$$10^{14} \text{ bacteria} = 100,000,000,000,000 \text{ bacteria} \quad \text{100 trillion}$$

Number of human cells in a human:

$$10^{13} \text{ cells} = 10,000,000,000,000 \text{ cells} \quad \text{10 trillion}$$

Number of bacteria on Earth:

$$5 \times 10^{30} \text{ bacteria}$$

$$= 5,000,000,000,000,000,000,000,000,000,000 \text{ bacteria}$$

✓ Checkpoint

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The diameter of a virus is less than 3×10^{-8} meter. Write this number in standard decimal notation and describe it in words. Which is larger, a bacteria or a virus?



EXAMPLE 4 Comparing Hard Drive Storage

You are buying a computer. For an additional \$80, you can get 1 terabyte of storage instead of 100 gigabytes of storage.

- How much more storage is that?
- A typical movie uses 1 gigabyte of storage. How many movies could you store with 1 terabyte of storage?

Computer Storage

1 megabyte = 10^6 bytes
 1 gigabyte = 10^9 bytes
 1 terabyte = 10^{12} bytes
 1 petabyte = 10^{15} bytes
 1 exabyte = 10^{18} bytes
 1 zettabyte = 10^{21} bytes
 1 yottabyte = 10^{24} bytes
 1 brontobyte = 10^{27} bytes
 1 geopbyte = 10^{30} bytes

**SOLUTION**

- 100 gigabytes of storage is

$$100 \times 10^9 = 100,000,000,000 \text{ bytes.} \quad \text{100 billion bytes}$$

- 1 terabyte of storage is

$$10^{12} = 1,000,000,000,000 \text{ bytes.} \quad \text{1 trillion bytes}$$

1 trillion is 10 times more than 100 billion. So, the additional \$80 will give you 10 times the amount of storage.

- A terabyte is 1000 gigabytes. So, you could store about 1000 movies with 1 terabyte of storage.

**✓ Checkpoint**

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Use the information in the paragraph below to find the storage of an Apple iPad.

Hello Zettabytes

The so-called digital universe has grown to 800,000 petabytes, or 0.8 zettabyte. A petabyte is a million gigabytes. That is equivalent to all the information that can be stored on 50 billion Apple iPads.



**Study Tip**

The best way to communicate measurements is to tell the reader how you obtained your numbers and to give some indication of the accuracy of the measurements.

Understanding Garbage In, Garbage Out

Humans tend to have unwarranted faith in numbers that are generated by calculators or by computers. When stating a number as an answer to a question, you should remember that the accuracy of the output is only as good as the accuracy of the input.

Significant Digit Rule

A general rule for writing numbers is to not list more **significant digits** in the output than you can guarantee in the input.

Examples: The number 1.26 has 3 significant digits. The number 0.034 has 2 significant digits. The number 25,400 has 3 significant digits.

EXAMPLE 5 Estimating the Accuracy of Measurements

Rewrite each statement using the number of significant digits that seems appropriate in the context of the statement. Explain your reasoning.

- In 2009, the population of Florida was 18,537,969 people.
- The total amount of a loan is \$3,546.28.
- The area of an apartment is 1891 square feet.

SOLUTION

- Taking a population census of a large region has many difficulties. Moreover, because 2009 is 9 years after an official census, this statement would be better phrased as, “In 2009, the population of Florida was about 18.5 million people.” This number has 3 significant digits.
- It is fair to assume that numbers dealing with banking or payroll are exact. So, this statement can remain as it is. All of the digits are significant.
- Without knowing who is making this statement, it is difficult to determine the accuracy. Even so, specifying the area of an apartment exactly within 1 square foot seems unrealistic. Suppose the apartment was measured to be a rectangle that is 31 feet by 61 feet. This would yield an area of $31 \times 61 = 1891$ square feet. If the measurements were off by only 0.1 foot, the actual area could range between

$$30.9 \times 60.9 = 1881.81 \quad \text{and} \quad 31.1 \times 61.1 = 1900.21.$$

It would be better to say that the area of the apartment is about 1900 square feet. This number has 2 significant digits.

 **Checkpoint**

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Rewrite each statement using the number of significant digits that seems appropriate in the context of the statement. Explain your reasoning.

- The weight of an athlete is 213.6 pounds.
- The record time for a 100-meter dash is 9.58 seconds.
- The distance between Earth and the Sun is 92,955,819 miles.



EXAMPLE 6 Writing Significant Digits

Here is a description of the Vietnam Veterans Memorial in Washington, D.C.

The Wall consists of the East Wall and the West Wall. The triangular walls are each 246.75 feet long and 10.1 feet tall where they meet at a 125-degree angle. The West Wall points to the Lincoln Memorial, and the East Wall points to the Washington Monument. Each Wall consists of 72 panels: 70 with names and 2 very small, blank panels at each end. There are 58,267 names listed on the Memorial. Approximately 1200 of these names are listed as missing.



Use the information in this description to estimate (a) the area of the Wall and (b) the size of the lettering used on the Wall. In each case, write your answer with only as many significant digits as you think are reasonable.

SOLUTION

- a. Find the area of each triangle.

$$\begin{aligned} \text{Area of each triangle} &= \frac{1}{2}bh \\ &= 0.5(246.75)(10.1) && \text{Base} = 246.75 \text{ ft} \\ & && \text{Height} = 10.1 \text{ ft} \\ &= 1246.0875 \text{ ft}^2 \end{aligned}$$

Doubling this amount gives a total of 2492.175 square feet. Because the height is given with only 3 significant digits, it is best to round the answer to the same. So, the most accurate you can be with the given description is to say that the Wall has an area of about 2490 square feet.

- b. Using an area of 2490 square feet, the average area per name is

$$\frac{58,267 \text{ names}}{2490 \text{ ft}^2} \approx 23.4 \text{ names per square foot.}$$

A square foot contains 144 square inches. So, you would have at most 6 square inches of space to carve each name. One reasonable solution is that the letters might be 1/2 inch high. This would leave 12 inches for the length of each name.

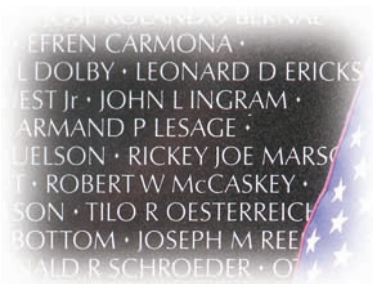
 **Checkpoint**

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Suppose you were planning the Wall. You want to list each name as

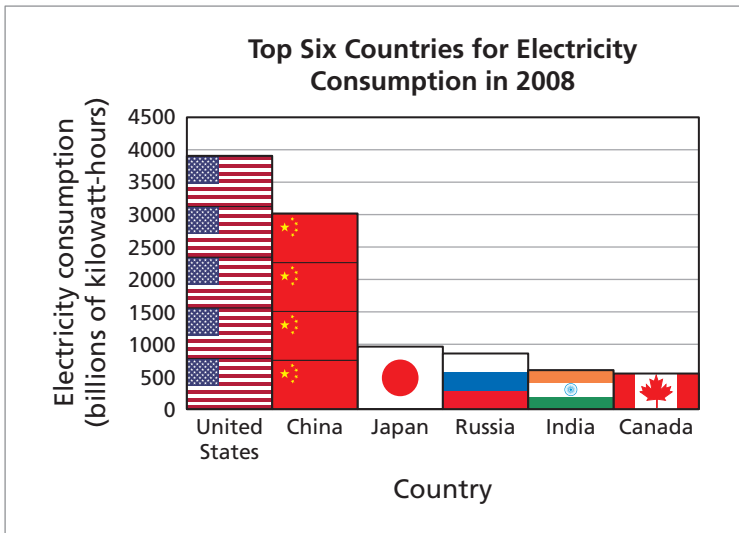
First Name Middle Initial Last Name.

How could you estimate the average number of characters used in the names? What is your estimate?



1.2 Exercises

DATA **Electricity Consumption** In Exercises 1–4, use the graph. Round your answer so it is reasonable for the context. (See Example 1.)



- Estimate the electricity consumption of the United States.
- Estimate the total electricity consumption of Japan, Russia, India, and Canada. Is it greater than or less than the electricity consumption of the United States?
- In 2008, the population of the United States was about 304 million. Estimate the amount of electricity consumed per person.
- In 2008, the population of Canada was about 33 million. Estimate the amount of electricity consumed per person. Which country consumed more electricity per person, Canada or the United States?

Refrigerator Costs Two refrigerator models and their annual electricity consumptions are shown. In Exercises 5–8, assume the price of electricity is \$0.1202 per kilowatt-hour (kWh). (See Example 2.)



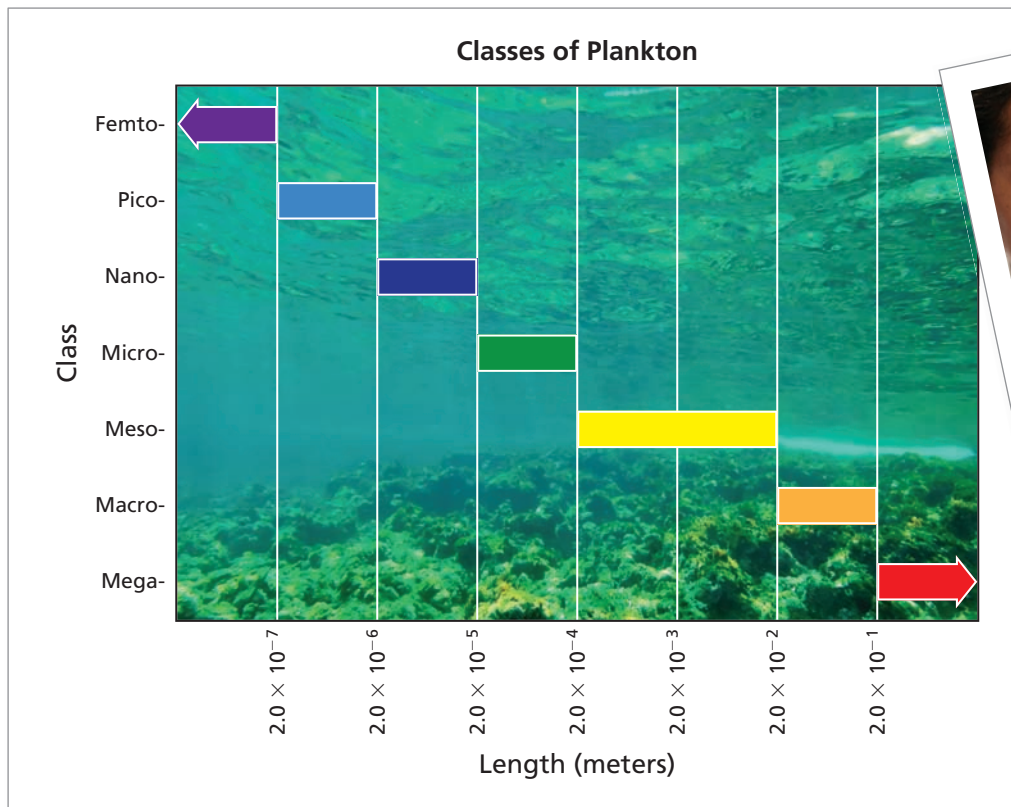
Top-freezer refrigerator
529 kWh/yr



Side-by-side refrigerator
634 kWh/yr

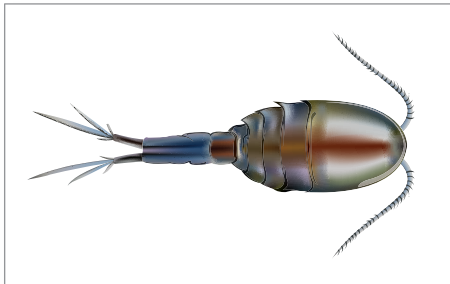
- Estimate the annual electricity cost of each model.
 - How much will you save in electric bills each year by buying the top-freezer model instead of the side-by-side model?
- A top-freezer model with an ice dispenser consumes 90 kilowatt-hours per year more than the standard top-freezer model.
 - Estimate the annual electricity cost of the top-freezer model with the ice dispenser.
 - How much does the ice dispenser add to the annual electricity cost of the top-freezer model?
- Your current refrigerator consumes 700 kilowatt-hours per year and should last about 7 more years.
 - How much will your current refrigerator cost you over the next 7 years?
 - How much will the top-freezer model cost you over the same time period?
 - You buy the top-freezer model for \$549.99. Will the difference in electricity costs over the next 7 years cover the price of the new refrigerator? Explain.
- You own the top-freezer model. Suppose the price of electricity decreases to \$0.12 per kilowatt-hour. How much will you save in electric bills each year?

Plankton In Exercises 9 and 10, write the length of the plankton in exponential notation. Then use the chart to classify the plankton. (See Examples 3 and 4.)

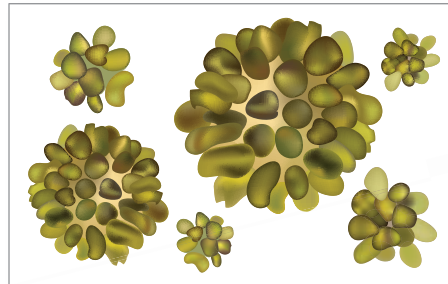


The term *plankton* comes from the Greek word *planktos*, which means "drifter."

9. Copepod: 0.0012 meter

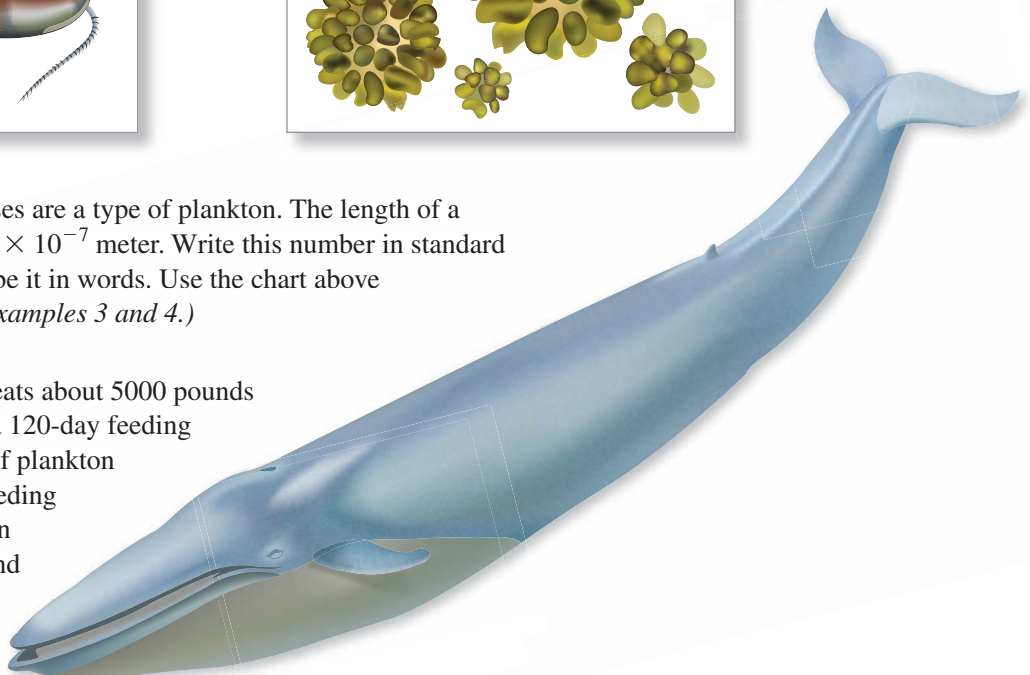


10. Golden algae: 0.0000091 meter



11. **Marine Virus** Marine viruses are a type of plankton. The length of a marine virus is less than 2.0×10^{-7} meter. Write this number in standard decimal notation and describe it in words. Use the chart above to classify the virus. (See Examples 3 and 4.)

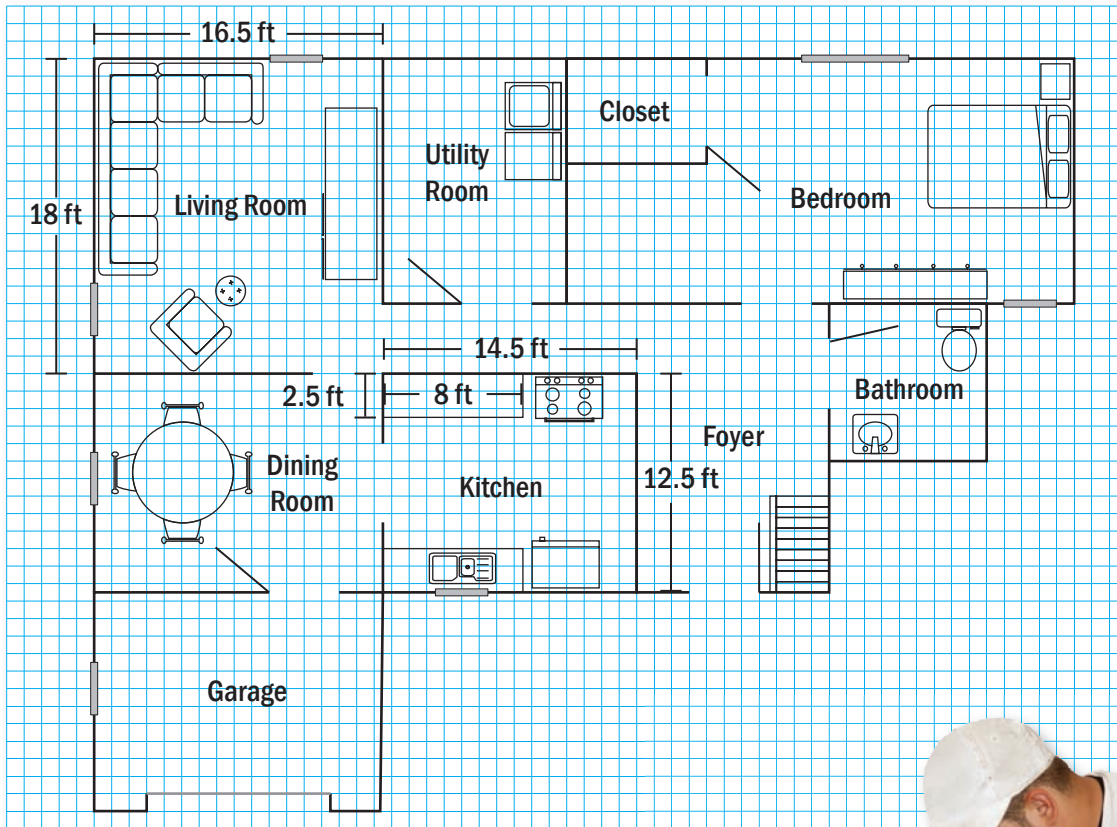
12. **Blue Whale** A blue whale eats about 5000 pounds of plankton per day during a 120-day feeding period. How many pounds of plankton does the whale eat in one feeding period? Write your answer in standard decimal notation and in exponential notation. (See Examples 3 and 4.)



Significant Digits In Exercises 13–16, rewrite each statement using the number of significant digits that seems appropriate in the context of the statement. Explain your reasoning. (See Example 5.)

13. In 2009, the population of Texas was 24,782,302 people. 14. The price of a house is \$239,900.
 15. The weight of a bag of sugar is 5.01 pounds. 16. The number of calories in a sandwich is 297.

Home Improvement In Exercises 17 and 18, use the floor plan. Write your answer with only as many significant digits as you think are reasonable. (See Example 6.)



17. You are carpeting the living room.
 a. How many square feet of carpet should you buy?
 b. The cost of the carpet including installation is \$3.49 per square foot. How much does it cost to carpet the living room?
18. You are replacing the linoleum in the kitchen.
 a. How many square feet of linoleum should you buy? Should you include the area of the two counters in your estimate? Explain.
 b. The cost of the linoleum including installation is \$1.25 per square foot. How much does it cost to replace the linoleum in the kitchen?

▶ Extending Concepts

Grocery Bill In Exercises 19 and 20, use rounding to decide whether the total is reasonable. Explain your reasoning.

19.

Bananas	\$2.79
Water	\$9.89
Pizza	\$5.99
Cereal	\$3.08
Lettuce	\$3.39
Tofu	\$6.48
	<hr/>
	\$31.62



20.

Oranges	\$1.89
Pie	\$3.49
Ice cream	\$3.22
Carrots	\$2.79
Dressing	\$2.96
Ketchup	\$2.46
	<hr/>
	\$23.11



Maps In Exercises 21–25, use a ruler and the scale on the map of Wyoming.

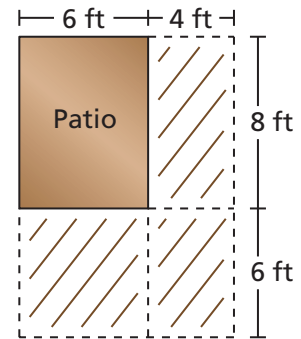
- Estimate the distance from Cheyenne to Gillette.
- You are planning a trip from Riverton to Gillette. You want to stop in Douglas along the way. Estimate the total distance of the trip.
- Your car can travel 274 miles on a tank of gas. Each tank contains 12.7 gallons. Estimate the fuel cost for a trip from Laramie to Evanston. Use a gas price of \$2.92 per gallon.



- Estimate the area of the portion of Yellowstone National Park that is in Wyoming.
- Describe different ways to find the distance between two cities.

1.1–1.2 Quiz

Patio You are expanding a patio as shown. In Exercises 1 and 2, use the diagram.



- Which of the following keystroke sequences is better for finding the area that you are adding to the patio? Explain your reasoning.
 - $(1 0 \times 1 4 - 6) \times 8 =$
 - $(1 0 \times 1 4) - (6 \times 8) =$
- You are laying 1-foot by 1-foot tiles. You estimate that it will take you 1 hour to lay 45 tiles. Estimate how long it will take you to tile the area that you are adding to the patio.

Population Density In Exercises 3–6, use the information given in the table.

- Population density is the number of people per unit of area.
 - Use the total area of Canada to estimate the country’s population density.
 - Land area is equal to total area minus water area. Use the land area of Canada to estimate the country’s population density.
 - Do you think population density should be defined in terms of the total area of a country or in terms of the land area? Explain your reasoning.
- Excluding Canada’s three northern territories, the population of Canada is about 34.0 million people, and the land area of Canada is about 5.5 million square kilometers.
 - Estimate the population density of Canada, excluding the three northern territories.
 - How does excluding the territories change your estimate of the population density? Explain your reasoning.
- The population of Toronto is about 2.5 million people, and the land area of Toronto is about 630 square kilometers.
 - Estimate the population density of Toronto.
 - How does your estimate for Toronto compare to your estimates for Canada?
- Do you think that population density is an accurate measure of a country’s population distribution? Explain your reasoning.



Canada	
Population (2010)	34,108,800
Total area	9,984,670 km ²
Water area	891,163 km ²



Math & Lunar Eclipses

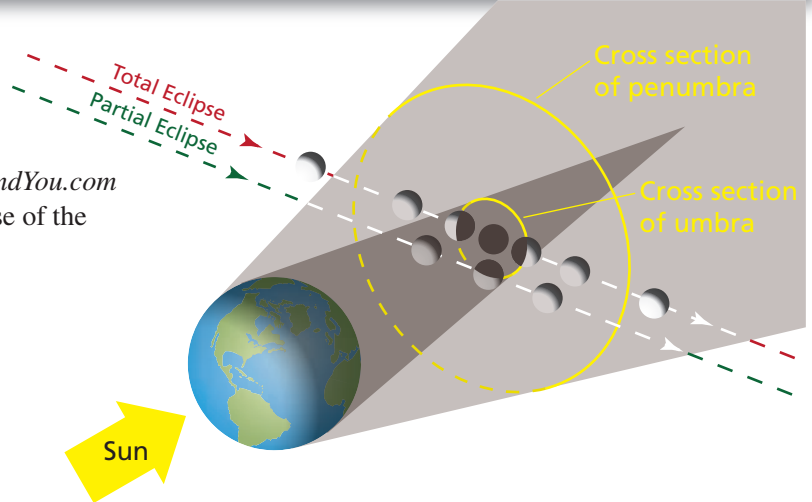
PROJECT: Finding Patterns in Nature

1. Use the *Lunar Eclipse Computer** at *Math.andYou.com* to find information about the next total eclipse of the moon in your area.

Eclipse:

City or Town Name:

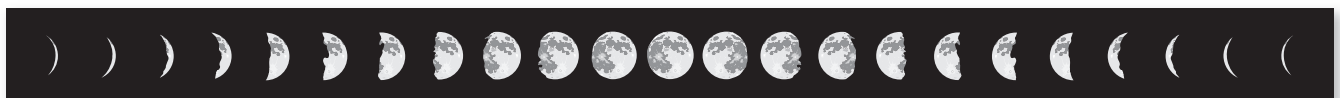
State or Territory:



Begin by entering your location information in the calculator. Then, press "Get Data" to obtain the information, as shown below.

			Moon's	
			Azimuth	Altitude
		h m	o	o
Moonrise	2010 Dec 20	15:51	56.9	----
Moon enters penumbra	2010 Dec 20	23:27.7	169.3	71.5
Moon enters umbra	2010 Dec 21	00:32.3	212.5	69.1
Moon enters totality	2010 Dec 21	01:40.4	241.3	59.9
Middle of eclipse	2010 Dec 21	02:17.0	251.2	53.8
Moon leaves totality	2010 Dec 21	02:53.6	259.1	47.4
Moon leaves umbra	2010 Dec 21	04:01.7	271.0	35.1
Moon leaves penumbra	2010 Dec 21	05:06.1	280.6	23.5
Moonset	2010 Dec 21	07:32	302.1	----

2. How long does a total lunar eclipse last?
3. How often does a total lunar eclipse occur? Explain your reasoning.
4. How has the time between lunar eclipses or between full moons influenced life on Earth? Include a discussion of tides, folklore, calendars, literature, and language.



*Provided by U.S. Naval Observatory

1.3 Using Percent

- ▶ Understand and find a percent of a number.
- ▶ Determine what percent one number is of another number.
- ▶ Use percent to represent change.

Finding a Percent of a Number

“Cent” implies 100, as in the word *century*. A **percent** is the number of parts per one hundred.

$$25\% = 0.25 = \frac{25}{100}$$

parts

per

one hundred

Study Tip

To rewrite a percent as a decimal, move the decimal point 2 places to the left.

$25\% = 0.25$

Finding a Percent of a Number

To find a percent of a number, write the percent as a decimal and multiply by the number.

Example: 25% of 60

$$0.25 \times 60 = 15$$

25% of 60

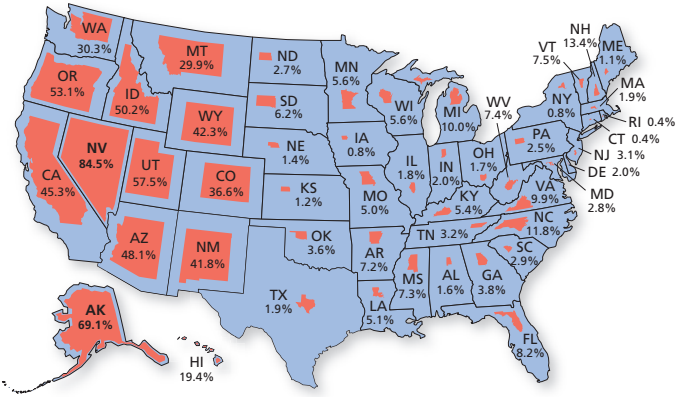
25% of 60

25% of 60

EXAMPLE 1 Finding a Percent of a Number

Percent of Land Owned by the Federal Government	
1. Nevada	84.5%
2. Alaska	69.1%
3. Utah	57.5%
4. Oregon	53.1%
5. Idaho	50.2%
6. Arizona	48.1%
7. California	45.3%
8. Wyoming	42.3%
9. New Mexico	41.8%
10. Colorado	36.6%

The map shows the percent of land in each state that is owned by the federal government. The land area of Nevada is 109,826 square miles. How many square miles of Nevada are owned by the federal government?



SOLUTION

Write 84.5% as a decimal.

$$0.845 \times 109,826 = 92,802.97$$

The federal government owns about 92,800 square miles of Nevada.

✔ Checkpoint

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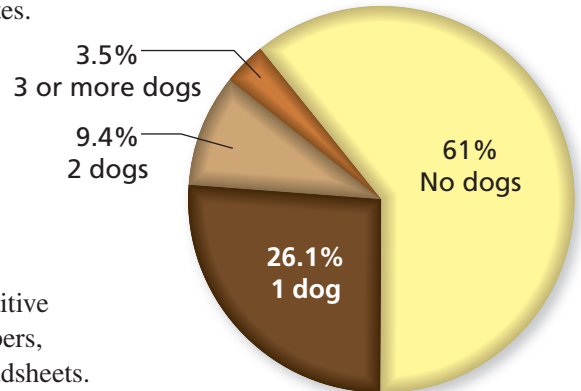
The land area of Alaska is 571,951 square miles. How many square miles of Alaska are owned by the federal government? Does the federal government own more land in Alaska or in Nevada?

EXAMPLE 2 Finding Percents



There are about 117 million households in the United States. Use the circle graph (also called a pie chart or pie graph) to estimate the number of pet dogs in the United States.

Dog Ownership in U.S. Households



SOLUTION

Problems like this, with repetitive steps and sums of large numbers, lend themselves well to spreadsheets.



$$117,000,000 \times 0.094 \times 2 = 21,996,000$$

	A	B	C	D
1	Households	Number of Dogs	Percent	Dogs
2	117,000,000	0	61.0%	0
3	117,000,000	1	26.1%	30,537,000
4	117,000,000	2	9.4%	21,996,000
5	117,000,000	5	3.5%	20,475,000
6			100.0%	73,008,000
7				
8				

Total percent

Total number of dogs

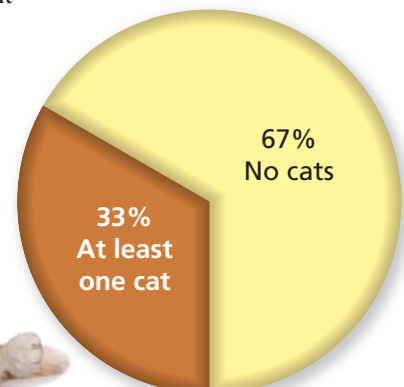
Notice that it was assumed that households with 3 or more dogs have an average of 5 dogs. This assumption affects the final answer. With this assumption and with the context of the problem, you cannot assume much accuracy in the answer. Perhaps an answer of “about 70 million dogs” is reasonable.

✓ Checkpoint

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There are about 117 million households in the United States. Thirty-three percent of the households own at least one cat. On average, these households have 2.45 cats. Estimate the number of pet cats in the United States.

Cat Ownership in U.S. Households



Using Percent to Compare Two Numbers

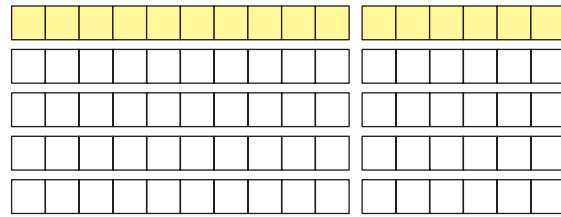
Comparing Two Numbers Using Percent

To find what percent a part is of a base, divide the part by the base.

Example:

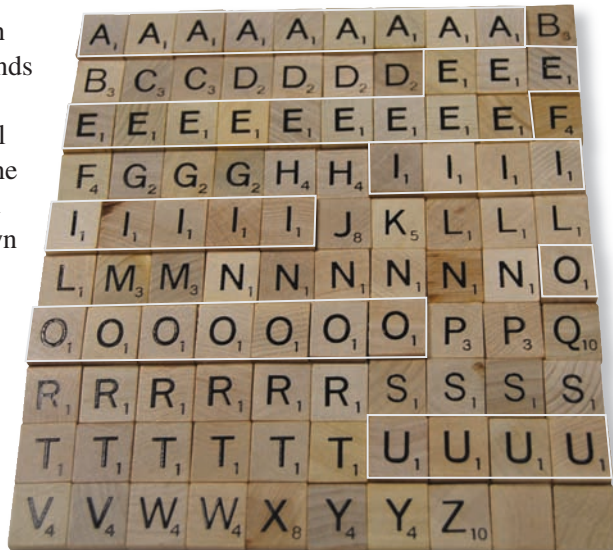
Part $\rightarrow \frac{16}{80} = 0.2 = 20\%$
 Base \rightarrow

16 is 20% or $\frac{1}{5}$ of 80



EXAMPLE 3 Comparing Two Numbers Using Percent

The number of tiles of each letter in SCRABBLE depends on the language. There are 100 letters in the traditional version of SCRABBLE. The distribution of the letters in the English version is shown at the right. In this version, what percent of the tiles are vowels?



SOLUTION

There are 9 A's, 12 E's, 9 I's, 8 O's, and 4 U's. That is a total of 42 vowels.

Part $\rightarrow \frac{42 \text{ tiles}}{100 \text{ tiles}} = 0.42 = 42\%$
 Base \rightarrow

42% of the tiles in the English version of SCRABBLE are vowels.

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The Spanish version of SCRABBLE has the following tiles.

- 2 blank tiles, 11 A's, 3 B's, 4 C's, 4 D's, 11 E's, 2 F's, 2 G's, 2 H's,
- 6 I's, 2 J's, 1 K, 4 L's, 1 LL, 3 M's, 5 N's, 1 Ñ, 8 O's, 2 P's, 1 Q,
- 4 R's, 1 RR, 7 S's, 4 T's, 6 U's, 2 V's, 1 W, 1 X, 1 Y, and 1 Z

What percent of these tiles are vowels?

EXAMPLE 4 Comparing Two Numbers Using Percent

A person's body fat percentage is the weight of the person's fat divided by the person's weight. The following ideal body fat percentages are from the American Council on Exercise.

Description	Women	Men
Athletes	14–20%	6–13%
Fitness	21–24%	14–17%
Average	25–31%	18–24%
Obese	32%+	25%+

Find the body fat percentage for the following people.

- A man who weighs 210 pounds with 44 pounds of fat
- A woman who weighs 145 pounds with 38 pounds of fat

SOLUTION

- A man who weighs 210 pounds with 44 pounds of fat:

$$\frac{\text{Fat weight}}{\text{Total weight}} = \frac{44 \text{ lb}}{210 \text{ lb}} \approx 0.2095 \approx 20.1\%$$

The man's body fat percentage is about 20%.

- A woman who weighs 145 pounds with 38 pounds of fat:

$$\frac{\text{Fat weight}}{\text{Total weight}} = \frac{38 \text{ lb}}{145 \text{ lb}} \approx 0.2621 \approx 26.2\%$$

The woman's body fat percentage is about 26%.

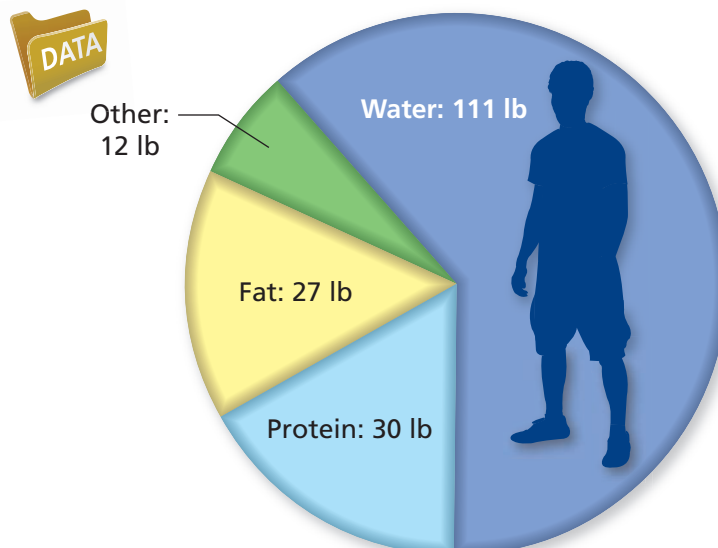
Study Tip

When you find a percent by dividing, the numerator and the denominator should have the same units.

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The circle graph represents a typical 180-pound man. Find the percent of each type of material.



Study Tip

In this section, we will only look at quantities that increase over time. In Chapter 4, we will look at quantities that increase or decrease over time.

Using Percent to Represent Change

In the first four examples of this section, percent is used to represent a part of a whole.

Example 1: Federally owned land is part of the *total* state land.

Example 2: Households with dogs are part of *all* households.

Example 3: Tiles with vowels are part of *all* tiles in SCRABBLE.

Example 4: Fat weight is part of the *total* weight of a person.

The last two examples in this section look at another common use of percent—that is, to describe the amount that a quantity changes over time.

EXAMPLE 5 Using Percent to Describe an Increase

Reword the statement so it uses a percent that is greater than 100%.

Life expectancy at birth in the United States in 1901 was 49 years. At the end of the century, it was 77 years, an increase of more than 50%.

SOLUTION

Compare 77 years to 49 years using division.

$$\frac{\text{Increased amount}}{\text{Original amount}} = \frac{77 \text{ years}}{49 \text{ years}} \approx 1.57 = 157\%$$

So, one way to rephrase the statement is as follows.

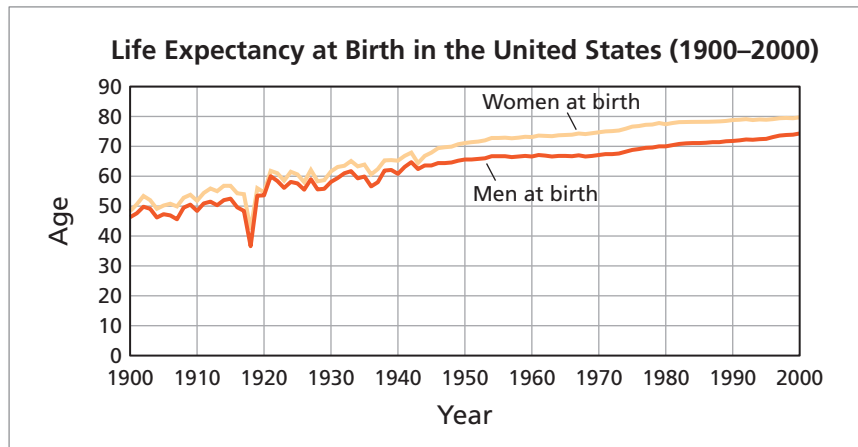
Life expectancy at birth in the United States in 1901 was 49 years. The life expectancy of 77 years in 2000 was 157% of the 1901 life expectancy.

✓ Checkpoint

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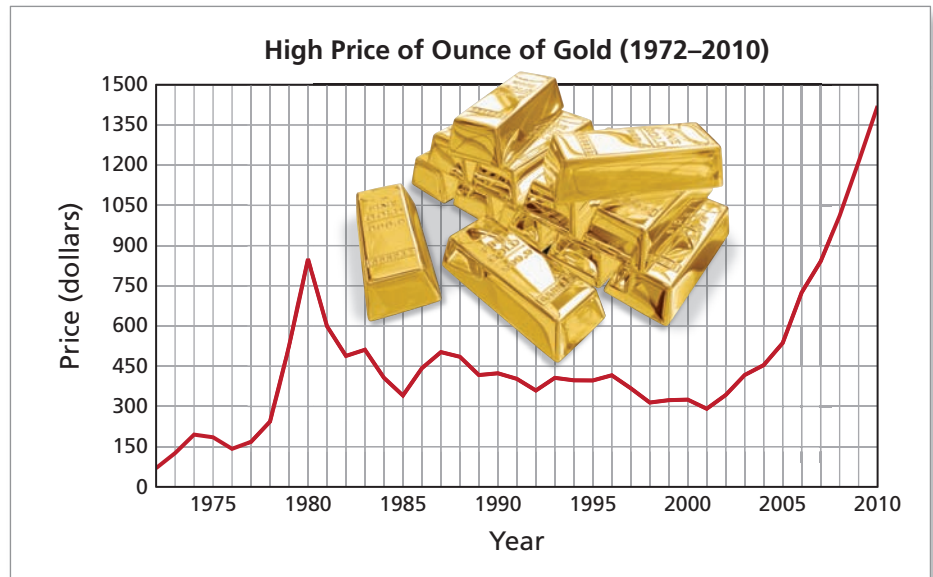
Use the line graph to write two paragraphs describing the change in women’s life expectancy at birth from 1900 to 2000.

- a. In the first paragraph, describe the change in life expectancy.
- b. In the second paragraph, compare the newer life expectancy to the older one.



EXAMPLE 6 Using Percent to Describe an Increase

The line graph shows the daily high price of gold per ounce. Use the graph to describe the change in the high price of gold (a) from 1980 to 2010 and (b) from 2000 to 2010.

**SOLUTION**

- a. Suppose you bought gold in 1980 for \$800 per ounce and sold it in 2010 for \$1200 per ounce.

$$\frac{\text{2010 price}}{\text{1980 price}} = \frac{\$1200}{\$800} = 1.5 = 150\%$$

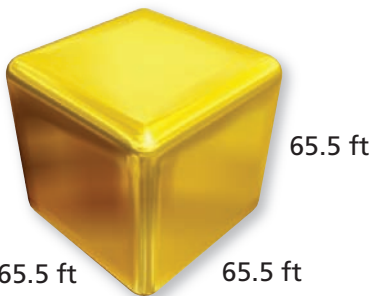
The 2010 price was 150% of the 1980 price. Another way of saying this is that your investment increased by 50%.

- b. Suppose you bought gold in 2000 for \$250 per ounce and sold it in 2010 for \$1250 per ounce.

$$\frac{\text{2010 price}}{\text{2000 price}} = \frac{\$1250}{\$250} = 5.0 = 500\%$$

The 2010 price was 500% of the 2000 price. Another way of saying this is that your investment increased by 400%. Be sure you see that the 500% is a result of comparing the two prices. If you compare only the *increase* in price to the original price, you obtain 400%.

$$\frac{\text{Price increase}}{\text{2000 price}} = \frac{\$1000}{\$250} = 4.0 = 400\%$$



All the gold that has been mined in human history would fit into a cube that is 65.5 feet on each side.

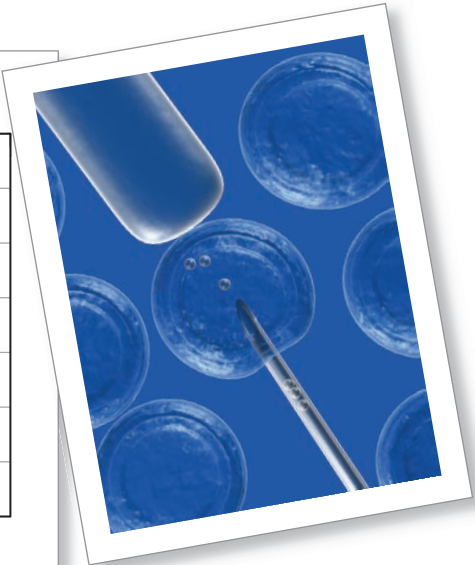
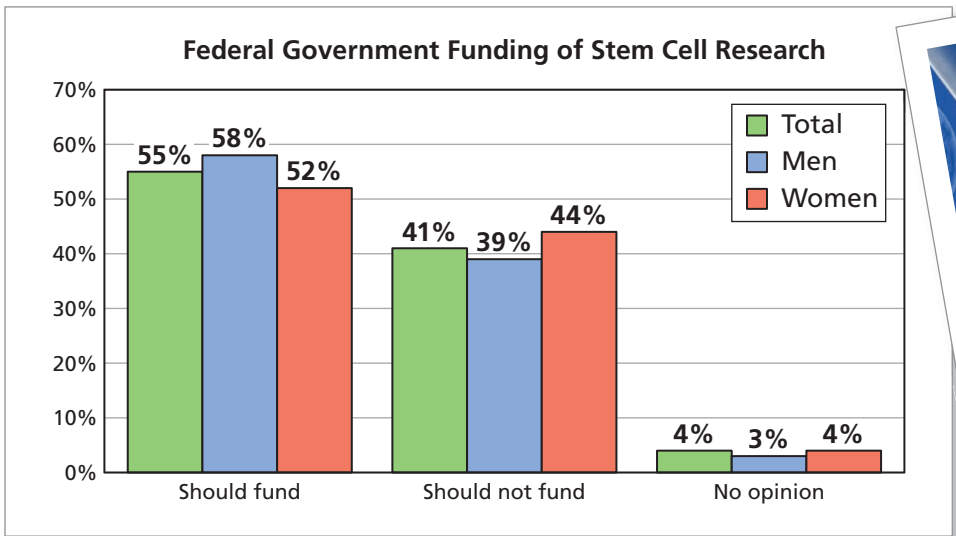
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Use the graph to describe the change in the high price of gold from 1975 to 2010.

1.3 Exercises

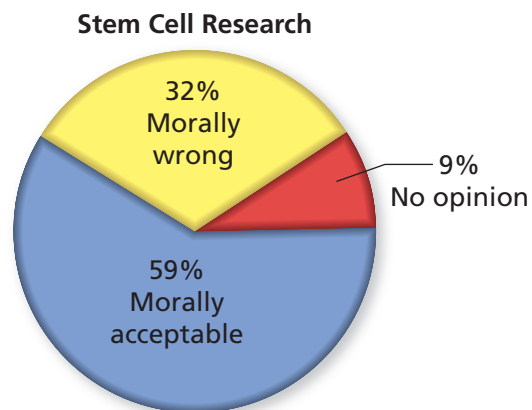
Stem Cell Research The bar graph shows the results of a poll of adult Americans about federal government funding of stem cell research. In Exercises 1–4, use the bar graph. (See Example 1.)



1. There are about 117,000,000 adult American men. About how many of these men think the federal government should fund stem cell research?
2. There are about 123,000,000 adult American women. About how many of these women think the federal government should fund stem cell research?
3. Describe two ways to find how many American adults have no opinion about federal government funding of stem cell research. Do both ways result in the same answer? Explain.
4. Are the results for men significantly different than the results for women? Explain.

Stem Cell Research The circle graph shows the results of a poll of adult Americans about the moral acceptability of stem cell research. In Exercises 5–8, use the circle graph. (See Example 2.)

5. In a city, 412,230 adults think that stem cell research is morally wrong. Estimate the population of the city.
6. How many adults from the city in Exercise 5 think stem cell research is morally acceptable?
7. Conduct a poll on stem cell research by asking at least 30 people for their opinion. Compare your results to the bar graph above and the circle graph.
8. Summarize the results of your poll in Exercise 7 graphically. Explain your choice of graph.





Skeletal System In Exercises 9–14, use the information below. (See Examples 3 and 4.)

The stapes (pronounced stay-peas) bone is the smallest bone in the body. It is sometimes called the stirrup bone because of its shape. It is located in the middle ear and is about the size of a grain of rice.



Bones in the Human Body

- Head – 29
- Hands – 38
- Wrists – 16
- Shoulders and arms – 10
- Hips and legs – 10
- Trunk – 51
- Feet – 38
- Ankles – 14

9. Of the hand bones, 28 are finger bones (14 phalanges in each hand). What percent of the hand bones are finger bones?

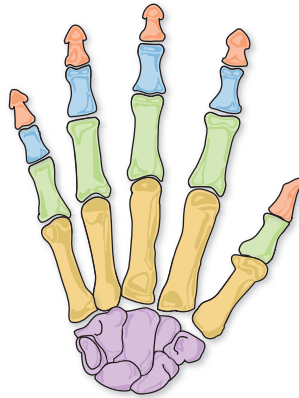
Distal phalanges

Intermediate phalanges

Proximal phalanges

Metacarpals

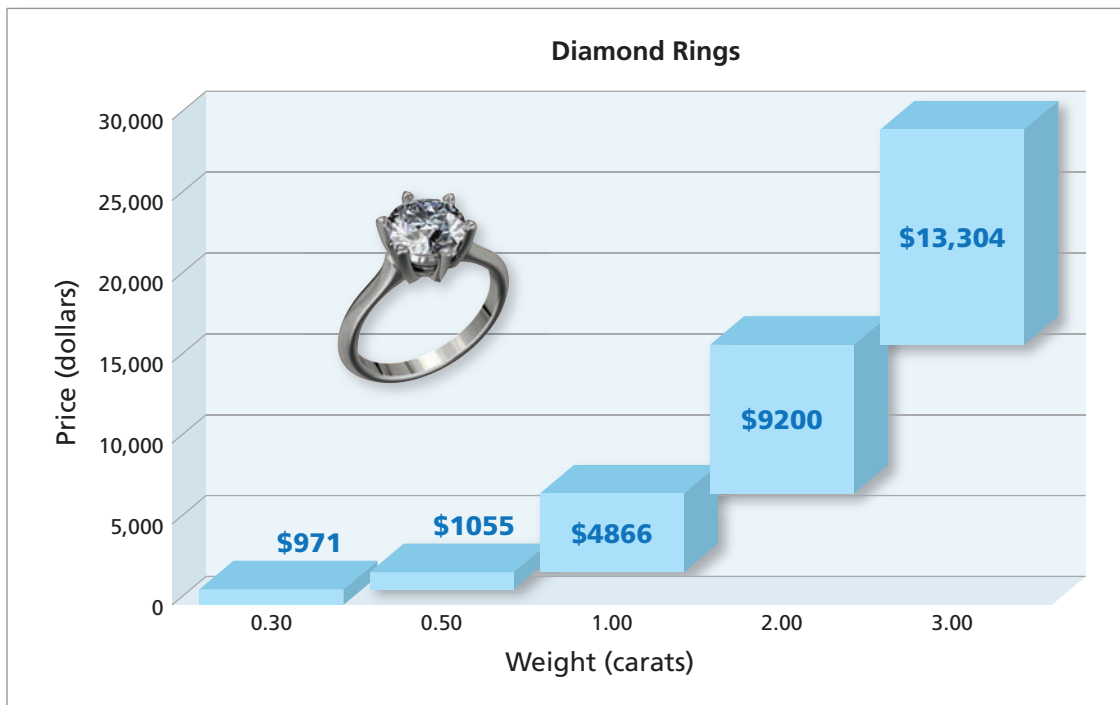
Carpals



10. Of the head bones, 14 are facial bones. What percent of the head bones are facial bones?
11. What percent of the bones in the human body are in the hips and legs?
12. What percent of the bones in the human body are in the trunk?
13. The skeletal system is a complex work of nature, with the hands and feet being the most intricate. What percent of the bones are in the hands, wrists, feet, and ankles?
14. The longest bone in the body is the thigh bone (femur). The average adult male has a femur that is 48 centimeters long. Use estimation and percent to compare the length of the stapes to the length of the femur in an average adult male.
15. **Skeleton** The skeleton of a 160-pound man weighs about 30 pounds. What percent of the man's weight is his skeleton? (See Examples 3 and 4.)
16. **Weight Gain** The man in Exercise 15 gains 30 pounds. What percent of the man's new weight is his skeleton? (See Examples 3 and 4.)



Diamond Rings The graph shows the price ranges for five weights of diamond rings at a jewelry store. In Exercises 17–24, use the graph. (See Examples 5 and 6.)



17. The jewelry store has a diamond ring listed at \$11,499. What is the carat weight of the diamond ring? Explain your reasoning.

18. What is the price range of a 1.00-carat diamond ring?

19. How much more does a 2.00-carat diamond ring cost than a 1.00-carat diamond ring?

20. Reword the statement so that it uses a percent that is greater than 200%.

A 0.50-carat diamond ring costs \$2026, which is about 108.7% more than the cost of a 0.30-carat diamond ring.

21. Is it true that a 0.50-carat diamond is more than 60% larger than a 0.30-carat diamond? Explain your reasoning.

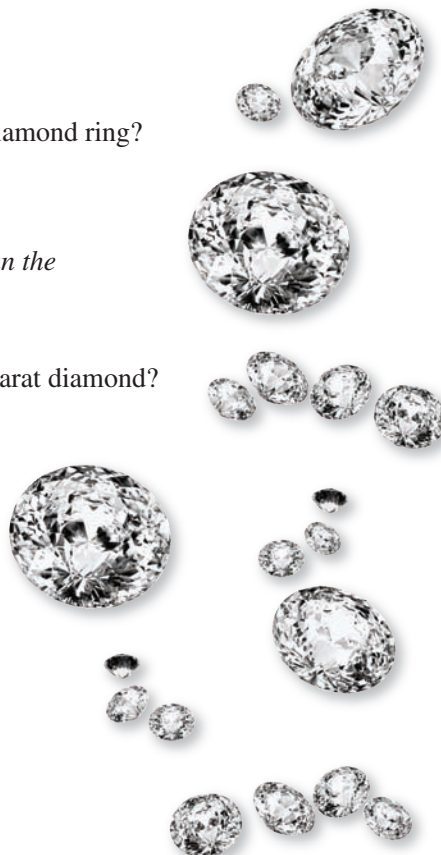
22. Use percent to describe the change in maximum price.

- a. from a 0.50-carat diamond ring to a 1.00-carat diamond ring
- b. from a 1.00-carat diamond ring to a 2.00-carat diamond ring
- c. from a 2.00-carat diamond ring to a 3.00-carat diamond ring

23. Use percent to describe the change in carat weight.

- a. from a 0.50-carat diamond ring to a 1.00-carat diamond ring
- b. from a 1.00-carat diamond ring to a 2.00-carat diamond ring
- c. from a 2.00-carat diamond ring to a 3.00-carat diamond ring

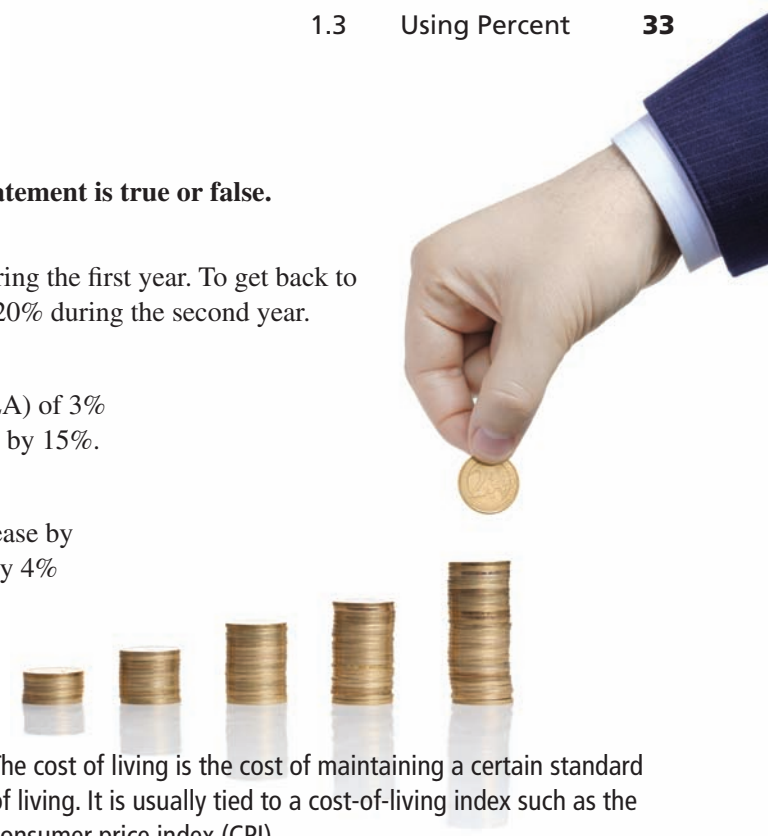
24. Describe the relationship between the carat weight and the price of a diamond ring.



▶ Extending Concepts

True or False In Exercises 25–28, decide whether the statement is true or false. Explain your reasoning.

25. You invest \$1250 in the stock market and lose 20% during the first year. To get back to your original investment, your portfolio needs to gain 20% during the second year.
26. At work, you receive a cost-of-living adjustment (COLA) of 3% each year. Over a 5-year period, your pay will increase by 15%.
27. Your property taxes increase by 5% one year and decrease by 1% the next year. Overall, your property taxes are up by 4% over 2 years.
28. Everything in a store is on sale for 40% off. You have a coupon for 10% off the purchase price. When you go to the counter to pay, you will receive 50% off the original price.

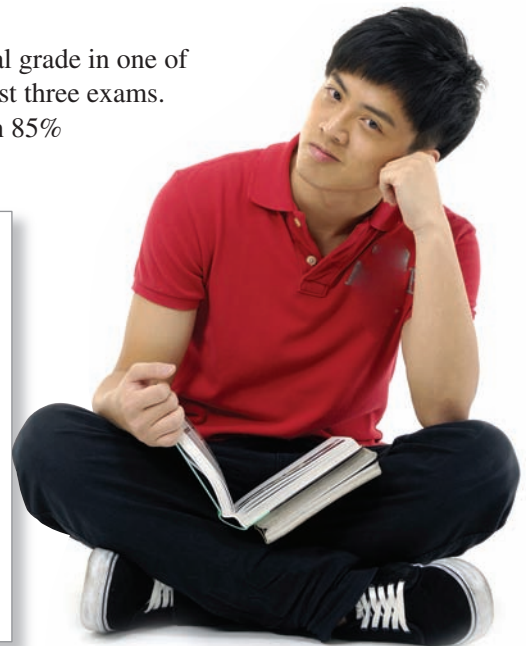
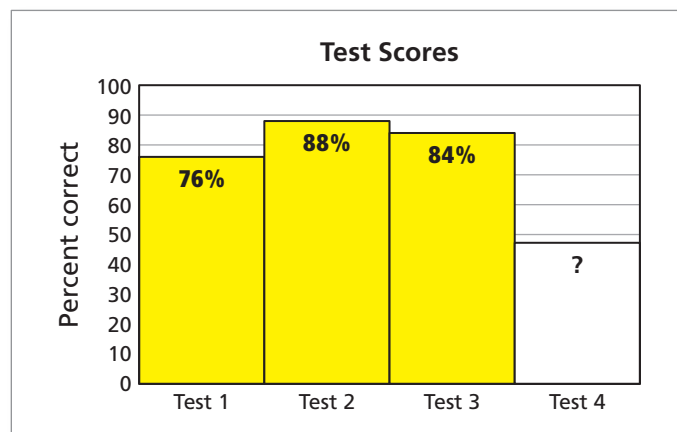


The cost of living is the cost of maintaining a certain standard of living. It is usually tied to a cost-of-living index such as the consumer price index (CPI).

29. **Savings Account** You open a savings account that has a 2.0% simple annual interest rate. At the end of the first year, you have earned \$30 in interest. No other transactions were posted to the account. What is the balance of the account at the end of the first year?
30. **Political Platform** As part of a political platform, a politician promises to cut spending by 110%. Is this possible? Explain your reasoning.



31. **Test Scores** Four exams are used to determine your final grade in one of your classes. The bar graph shows your scores on the first three exams. What percent do you need on the last exam to receive an 85% in the class?



32. **Profit Sharing** Your company currently allocates 5% of the company profits each year to a profit sharing plan for the employees. The board of directors decides to increase the amount allocated by 10%. Does this mean the new policy is to allocate 5.5% or 15% to the profit sharing plan each year? Explain your reasoning.

1.4 Units & Conversions

- ▶ Use unit analysis to “balance” both sides of a formula.
- ▶ Convert within a given system of measure.
- ▶ Convert between different systems of measure.

Using Unit Analysis when Evaluating Formulas

Everything you measure has a number with some sort of **unit of measure** attached. The unit can be feet, pounds, dollars per hour, people, degrees, or any of countless other ways in which real-life objects are measured.

Study Tip

Many people have found that unit analysis is the most useful “trick” they were ever taught in math. Using it and understanding it can open your eyes to all sorts of math that appears mysterious without it.

Unit Analysis

Unit analysis is the process of changing from one unit of measure to another. Change occurs when units are multiplied or divided. Addition and subtraction always preserve units.

Examples: $2 \text{ ft} + 3 \text{ ft} = 5 \text{ ft}$	Addition preserves units.
$8 \text{ oz} - 2 \text{ oz} = 6 \text{ oz}$	Subtraction preserves units.
$\left(18 \frac{\$}{\text{hr}}\right)(10 \text{ hr}) = \180	Multiplication changes units.
$\frac{320 \text{ mi}}{8 \text{ gal}} = 40 \frac{\text{mi}}{\text{gal}}$	Division changes units.

EXAMPLE 1 Using Unit Analysis

In 2010, the average hourly billing rate for attorneys in the United States was \$413 per hour for litigation, \$302 per hour for labor or employee cases, and \$294 per hour for real estate cases.

- a. What would you expect to pay for 1200 hours of litigation?
- b. You paid \$2850 for 9.5 hours of legal work on a mortgage. What hourly rate did your attorney charge?
- c. Your attorney represented you for 25% contingency on a personal injury case in which you were awarded \$600,000. Your attorney spent 1000 hours on the case. What was the hourly billing rate?

SOLUTION

a. $\left(413 \frac{\$}{\text{hr}}\right)(1200 \text{ hr}) = \$495,600$	b. $\frac{\$2850}{9.5 \text{ hr}} = 300 \frac{\$}{\text{hr}}$
c. $\frac{25\% \text{ of } \$600,000}{1000 \text{ hr}} = \frac{\$150,000}{1000 \text{ hr}} = 150 \frac{\$}{\text{hr}}$	

✓ Checkpoint

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An attorney at a law firm billed an average of 55 hours in litigation fees per week for a year. Estimate the amount of revenue the attorney brought into the law firm during the year.



EXAMPLE 2 Using Unit Analysis



The giant panda was listed as endangered in 1990.

It is estimated that about 125 species of birds and 60 species of mammals have become extinct since 1600. There are approximately 1000 species of birds and mammals that are currently facing extinction. Including all types of plant and animal species, the number of endangered species is around 20,000.



The Endangered Species Coalition works to protect endangered species and their habitats.

- a. Estimate the number of species of birds that will become extinct in the next 100 years.
- b. Estimate the number of years in which 40 more species of mammals will have become extinct.

SOLUTION

This estimate is, of course, quite primitive because it is based only on the rate of extinction during the 410 years from 1600 to 2010.

a. $\left(\frac{125 \text{ species}}{410 \text{ years}}\right)(100 \text{ years}) \approx 30.49 \text{ species}$

So, you could estimate that 30 species of birds will become extinct during the next 100 years.

- b. This is the most difficult type of question in unit analysis. You need to perform an operation so that the resulting units are years. To do this, you can use the *Invert and Multiply Rule* for dividing by a fraction.

$$\frac{40 \text{ species}}{60 \text{ species}} \div \frac{410 \text{ years}}{60 \text{ species}} = (40 \text{ species}) \left(\frac{410 \text{ years}}{60 \text{ species}}\right) \approx 273.33 \text{ years}$$

Invert the denominator and multiply.

So, you could estimate that it might take about 270 years for an additional 40 species of mammals to become extinct.

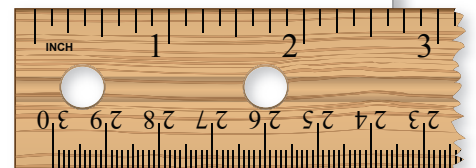


The bald eagle was once listed as an endangered species. In 2007, it was removed from the list due to success in protecting the species.

Here is a simple example of dividing by a fraction. How many one-eighth inches are in 2 inches?

$$\frac{2 \text{ in.}}{\frac{1}{8} \text{ in.}} = 2 \left(\frac{8}{1}\right) = 16$$

Invert the denominator and multiply.



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- c. Estimate the number of species of mammals that will become extinct in the next 100 years.
- d. Estimate the number of years in which 40 more species of birds will have become extinct.

Converting Units within a Given System of Measure

Converting Units

To **convert** from one type of unit to another, multiply by a convenient form of the number one.

One

$$\text{Example: } 2 \text{ ft} = 2 \cancel{\text{ft}} \left(\frac{12 \text{ in.}}{1 \cancel{\text{ft}}} \right) = 24 \text{ in.}$$



Common Unit Conversions

	U.S. Customary	Metric
Distance:	1 mi = 5280 ft 1 yd = 3 ft 1 ft = 12 in.	1 km = 1000 m 1 m = 100 cm 1 cm = 10 mm
Volume/Capacity:	1 gal = 4 qt 1 qt = 4 cups 1 cup = 8 fl oz	1 L = 1000 mL
Weight/Mass:	1 lb = 16 oz	1 kg = 1000 g

EXAMPLE 3 Converting Units

How many seconds are in a calendar year of 365 days?

SOLUTION

At first, this question might seem overwhelming. But you can find the answer by simply multiplying by three carefully chosen forms of the number one.

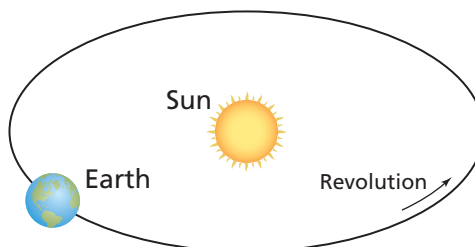
$$\begin{aligned}
 365 \text{ days} &= (365 \cancel{\text{days}}) \left(\frac{24 \cancel{\text{hr}}}{1 \cancel{\text{day}}} \right) \left(\frac{60 \cancel{\text{min}}}{1 \cancel{\text{hr}}} \right) \left(\frac{60 \text{ sec}}{1 \cancel{\text{min}}} \right) \\
 &= 31,536,000 \text{ sec}
 \end{aligned}$$

There are 31,536,000 seconds in a calendar year of 365 days.

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A *solar year* is the time it takes Earth to complete one revolution of its orbit about the Sun. A solar year is 365.24218967 days. How many more seconds are in a solar year than in a calendar year?

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Study Tip

A day is the length of time it takes Earth to complete one revolution about its axis. The fact that a solar year is not a whole number of days has perplexed calendar makers for many centuries.

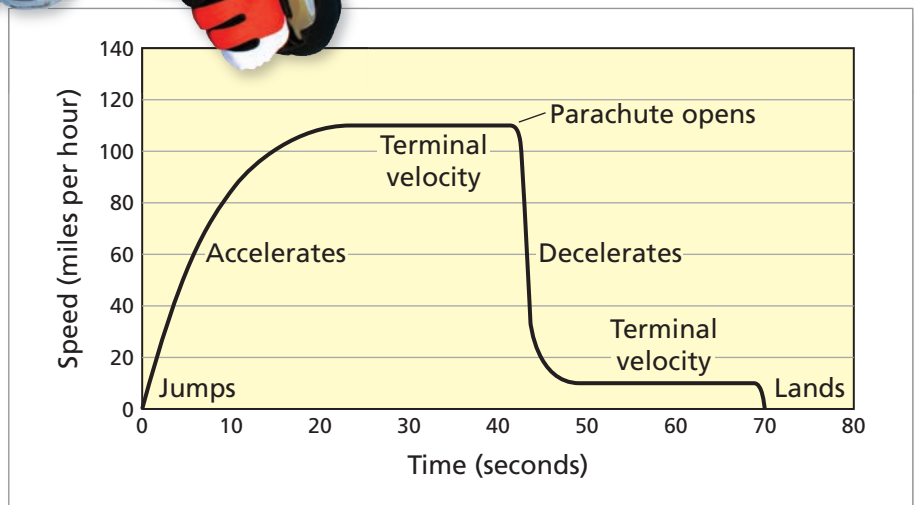
EXAMPLE 4 Converting Units

When an object falls, it accelerates. As its speed increases, the air resistance increases.

Eventually, the air resistance will offset the weight of the object, and the object will reach “terminal velocity.”

Use the graph to find each velocity in *feet per second*.

- Terminal velocity *before* the parachute opens—during free fall
- Terminal velocity *after* the parachute opens

**Study Tip**

Notice the difference in the “form of one” used in the solutions.

Example 3: $\frac{60 \text{ sec}}{1 \text{ min}}$

Example 4: $\frac{1 \text{ min}}{60 \text{ sec}}$

The form used depends on the units you are trying to obtain in your answer.

SOLUTION

- a. From the graph, the free-fall terminal velocity is about 110 miles per hour.

$$110 \frac{\text{mi}}{\text{hr}} = \left(110 \frac{\cancel{\text{mi}}}{\cancel{\text{hr}}} \right) \left(\frac{5280 \text{ ft}}{1 \cancel{\text{mi}}} \right) \left(\frac{1 \cancel{\text{hr}}}{60 \cancel{\text{min}}} \right) \left(\frac{1 \cancel{\text{min}}}{60 \text{ sec}} \right) \approx 161.33 \frac{\text{ft}}{\text{sec}}$$

The free-fall terminal velocity is about 161 feet per second.

- b. The parachute terminal velocity is about 10 miles per hour.

$$10 \frac{\text{mi}}{\text{hr}} = \left(10 \frac{\cancel{\text{mi}}}{\cancel{\text{hr}}} \right) \left(\frac{5280 \text{ ft}}{1 \cancel{\text{mi}}} \right) \left(\frac{1 \cancel{\text{hr}}}{60 \cancel{\text{min}}} \right) \left(\frac{1 \cancel{\text{min}}}{60 \text{ sec}} \right) \approx 14.66 \frac{\text{ft}}{\text{sec}}$$

The parachute terminal velocity is about 15 feet per second.

✓ Checkpoint

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- Estimate the time the parachutist spends in free fall. Explain your reasoning.
- Estimate the time the parachutist spends with the parachute open. Explain your reasoning.

Study Tip

Quick approximations:

$1 \text{ mi} \approx \frac{8}{5} \text{ km}$

$F \approx 2C + 30$

$1 \text{ gal} \approx 4 \text{ L}$

$1 \text{ lb} \approx \frac{2}{5} \text{ kg}$

Converting Units Between Different Systems

Common Unit Conversions	
Distance:	Temperature:
$1 \text{ mi} \approx 1.61 \text{ km}$	$F = \frac{9}{5}C + 32$
$1 \text{ in.} = 2.54 \text{ cm}$	Weight/Mass:
Volume/Capacity:	$1 \text{ lb} \approx 0.45 \text{ kg}$
$1 \text{ gal} \approx 3.79 \text{ L}$	

EXAMPLE 5 Converting Units



You are driving on a Canadian highway. The speed limit is 100 kilometers per hour. You figure that you can drive 10 kilometers per hour over the limit without getting a ticket. Your speedometer only shows miles per hour.

- What is the maximum speed you should be traveling in miles per hour?
- Following your “rule,” what is the maximum speed you should be traveling in miles per hour to avoid getting a speeding ticket?

SOLUTION

a. $100 \text{ km} = 100 \text{ km} \left(\frac{1 \text{ mi}}{1.61 \text{ km}} \right) \approx 62 \text{ mi}$

The speed limit is about 62 miles per hour.

b. $110 \text{ km} = 110 \text{ km} \left(\frac{1 \text{ mi}}{1.61 \text{ km}} \right) \approx 68 \text{ mi}$

Your “rule” indicates that you can drive up to 68 miles per hour without getting a speeding ticket.

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While in Canada, you stop at a gas station. You have heard that gas prices in Canada are considerably more expensive than in the United States.

- From the sign at the left, does this seem to be true?
- To answer part (c), what conversion should you consider in addition to the conversion between gallons and liters?

EXAMPLE 6 Converting Temperature Units

You remember hearing that there is one temperature that has the same degree measure in the Fahrenheit and Celsius scales. You cannot remember what the temperature is. How can you find it?

SOLUTION

One way is to use algebra. You could let $C = F$ and solve for F in the equation

$$F = \frac{9}{5}F + 32.$$

$$0 = \frac{9}{5}F - F + 32 \quad \text{Subtract } F \text{ from each side.}$$

$$0 = \frac{4}{5}F + 32 \quad \text{Combine like terms.}$$

$$-32 = \frac{4}{5}F \quad \text{Subtract 32 from each side.}$$

$$-160 = 4F \quad \text{Multiply each side by 5.}$$

$$-40 = F \quad \text{Divide each side by 4.}$$

But you may have forgotten how to solve this equation. Another instructive way to find the temperature is to use a spreadsheet.



	A	B	C
1	Celsius	Fahrenheit	
2	100	212	
3	90	194	
4	80	176	
5	70	158	
6	60	140	
7	50	122	
8	40	104	
9	30	86	
10	20	68	
11	10	50	
12	0	32	
13	-10	14	
14	-20	-4	
15	-30	-22	
16	-40	-40	
17	-50	-58	

$(9/5) * A2 + 32$



The temperature -40 degrees is the same on both scales.

✓ Checkpoint

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You are staying at a hotel in Canada. Your room feels cold, and you notice that the temperature is set at 20°C .

- What is the room temperature in degrees Fahrenheit?
- What should you set the temperature at to obtain a temperature of 77°F ?

1.4 Exercises

Online Shopping The table shows an online store's shipping rates. In Exercises 1–6, use the table. (See *Examples 1 and 2.*)



Delivery Time	Shipping Rate
3–5 days	\$0.49/lb
2 days	\$0.89/lb
1 day	\$1.89/lb

1. You order a 5-pound textbook with 3–5 day shipping. What is the shipping fee?
2. You order a 330-pound weight set with 2-day shipping. What is the shipping fee?
3. You order a 26-pound television. Suppose you use 3–5 day shipping instead of 2-day shipping. How much will you save?

4. You order a 119-pound couch. Suppose you use 1-day shipping instead of 3–5 day shipping. How much more will you pay?
5. You wait until the last minute to buy a 159-pound treadmill for a birthday present. The online price of the treadmill is \$110 less than the price of the same treadmill at a local fitness store. You need the treadmill by tomorrow, so you will have to pay for 1-day shipping.
 - a. Estimate the shipping fee for the treadmill.
 - b. Will it cost more to order the treadmill online with 1-day shipping or to buy the treadmill at the local fitness store?
 - c. Does your answer to part (b) change if you had an extra day to place your order? What if you had an extra week?
6. You recently purchased a pair of headphones online. The shipping fee was \$12.24. You believe that you were overcharged for 3–5 day shipping, but the weight of the headphones is not listed on the receipt.
 - a. How much do the headphones have to weigh for the fee to be correct? Is the weight reasonable?
 - b. If the weight is not reasonable, write an email to the store explaining the error and asking for a refund.



Travel Time The road sign shows your distance to several nearby cities in miles. In Exercises 7 and 8, use the road sign. (See Example 3.)

7. You are driving at 45 miles per hour. Which of the following expressions should you use to determine how long it will take to get to Ventura? How long will it take?

a. $54 \text{ mi} \times \frac{45 \text{ mi}}{1 \text{ hr}} \times \frac{60 \text{ min}}{1 \text{ hr}} = \square$

b. $54 \text{ mi} \times \frac{1 \text{ hr}}{45 \text{ mi}} \times \frac{60 \text{ min}}{1 \text{ hr}} = \square$

c. $54 \text{ mi} \times \frac{45 \text{ mi}}{1 \text{ hr}} \times \frac{1 \text{ hr}}{60 \text{ min}} = \square$

d. $54 \text{ mi} \times \frac{1 \text{ hr}}{45 \text{ mi}} \times \frac{1 \text{ hr}}{60 \text{ min}} = \square$



8. You are driving at 65 miles per hour. How long will it take to get to Los Angeles?



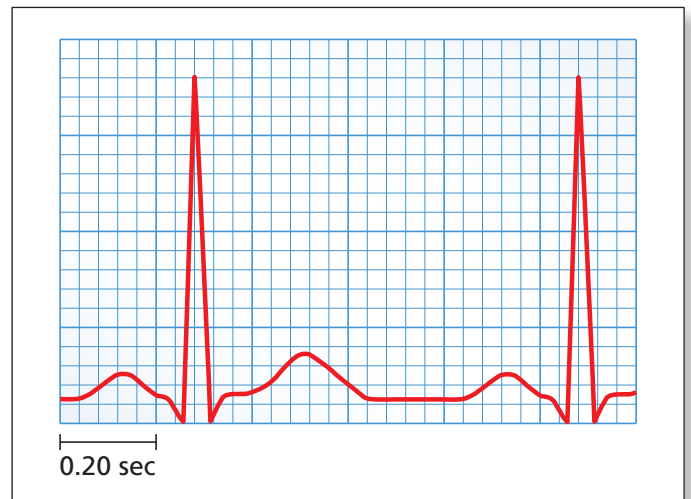
A human heart weighs about 10 ounces. On average, it beats about 70 times per minute and pumps about 2.5 fluid ounces of blood with each beat.

Human Heart In Exercises 9–12, use the information. (See Examples 3 and 4.)

9. How much does a heart weigh in pounds?
 10. A human brain weighs about 3 pounds. How many ounces heavier is a brain than a heart?
 11. How many quarts of blood does a heart pump in 1 minute?
 12. How many gallons of blood does a heart pump in 1 day?

Heart Rate A patient’s heart rate can be determined from an electrocardiogram (EKG). The time between two peaks on the EKG represents one heartbeat. In Exercises 13 and 14, use the EKG. (See Example 4.)

13. Ringo Starr sang a song that had the words “In a heartbeat, I’ll be by your side.” What is the length of one heartbeat?
 14. What is the heart rate of the person whose EKG is shown? Is this heart rate normal?



Seven Summits The map shows the height of the highest mountain on each of Earth's seven continents. Climbing all seven mountains, also known as the Seven Summits, is considered a challenge in mountaineering. In Exercises 15–20, use the information on the map. (See Examples 5 and 6.)



15. Determine the height of Mount McKinley in feet.
16. Order the heights of the Seven Summits from highest to lowest.
17. You are climbing Mount Everest. After 1 hour, you have climbed 183 meters. Find your rate of ascent in feet per minute.
18. The original Seven Summits included Mount Kosciuszko in mainland Australia instead of Mount Carstensz. Mount Kosciuszko is about 2228 meters high. Is it higher or lower than Mount Carstensz? Explain.
19. At the summit of Mount Everest, water boils at 69°C .
 - a. Convert this temperature to degrees Fahrenheit.
 - b. Could you boil a potato at the summit until it is cooked? Explain.
20. For every 1000 feet of elevation, the air temperature drops about 6.5°C . Assume the temperature at the foot of Mount Kilimanjaro is 23°C . Estimate the temperature at the summit in degrees Fahrenheit.



The Seven Summits on the map are also known as the Messner List, named after mountaineer Reinhold Messner.

► Extending Concepts

Exchange Rates The table shows a sample of currency exchange rates for November 2010. In Exercises 21–24, use the table.

Currency	From U.S. Dollars to Foreign Currency	From Foreign Currency to U.S. Dollars
British pound	$\$1 = \text{£}0.6325$	$\text{£}1 = \$1.5813$
European euro	$\$1 = \text{€}0.7465$	$\text{€}1 = \$1.3396$
Canadian dollar	$\$1 = \text{C}\1.0239	$\text{C}\$1 = \0.9767
Mexican peso	$\$1 = \text{Mex}\12.4823	$\text{Mex}\$1 = \0.0801
Japanese yen	$\$1 = \text{¥}83.065$	$\text{¥}1 = \$0.012$
Indian rupee	$\$1 = \text{₹}45.81$	$\text{₹}1 = \$0.0218$



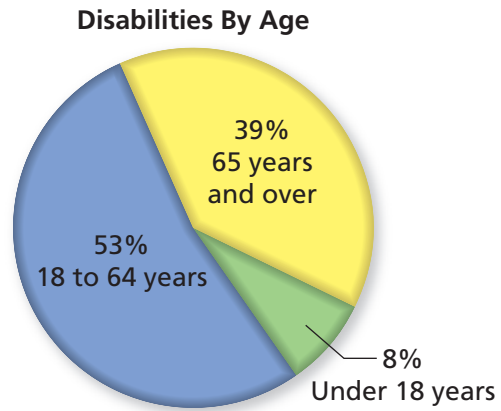
21. You order a product from a British website. The list price is £223. How much does the product cost in U.S. dollars?
22. In Germany, an airline charges an extra €10 for each additional kilogram of carry-on luggage over 10 kilograms. Your carry-on luggage weighs 26.5 pounds. How much extra can you expect to pay in U.S. dollars?
23. A gas station in Mexico City charges Mex\$7.73 for a liter of gasoline. Your car's 23-gallon tank is almost empty. How much will it cost in U.S. dollars to fill the tank?
24. You are in the United States and plan to travel to several other countries. You have \$3000 in cash.
 - a. Your first destination is Canada. Convert your cash to Canadian dollars.
 - b. You leave Canada for Great Britain with C\$2800. Convert this to British pounds.
 - c. You depart from Great Britain for Spain with 1400 British pounds. Convert this to European euros.
 - d. Your next stop is India. After your time in Spain, you have 1000 euros. Convert this to Indian rupees.
 - e. From India, you leave for Japan. You have 50,000 rupees after your time in India. Convert this to Japanese yen.
 - f. You return to the United States with 80,000 Japanese yen. Convert this to U.S. dollars. How much money did you spend total?



1.3–1.4 Quiz

Disabilities The circle graph shows the age categories of people with disabilities in the United States. In Exercises 1–4, use the circle graph.

- There are about 36 million people with disabilities in the United States. How many of the people with disabilities are 65 years old or older?
- The population of the United States is about 310 million. Estimate the percent of the general population that is disabled.
- People who are 65 years old and over only make up about 13% of the general population. Compare this to the percent of the disabled population for this group. Why do you think there is a discrepancy?
- Compare the percent for people under 18 years old to the percents for the other categories. Why do you think the percent for people under 18 years old is so small?

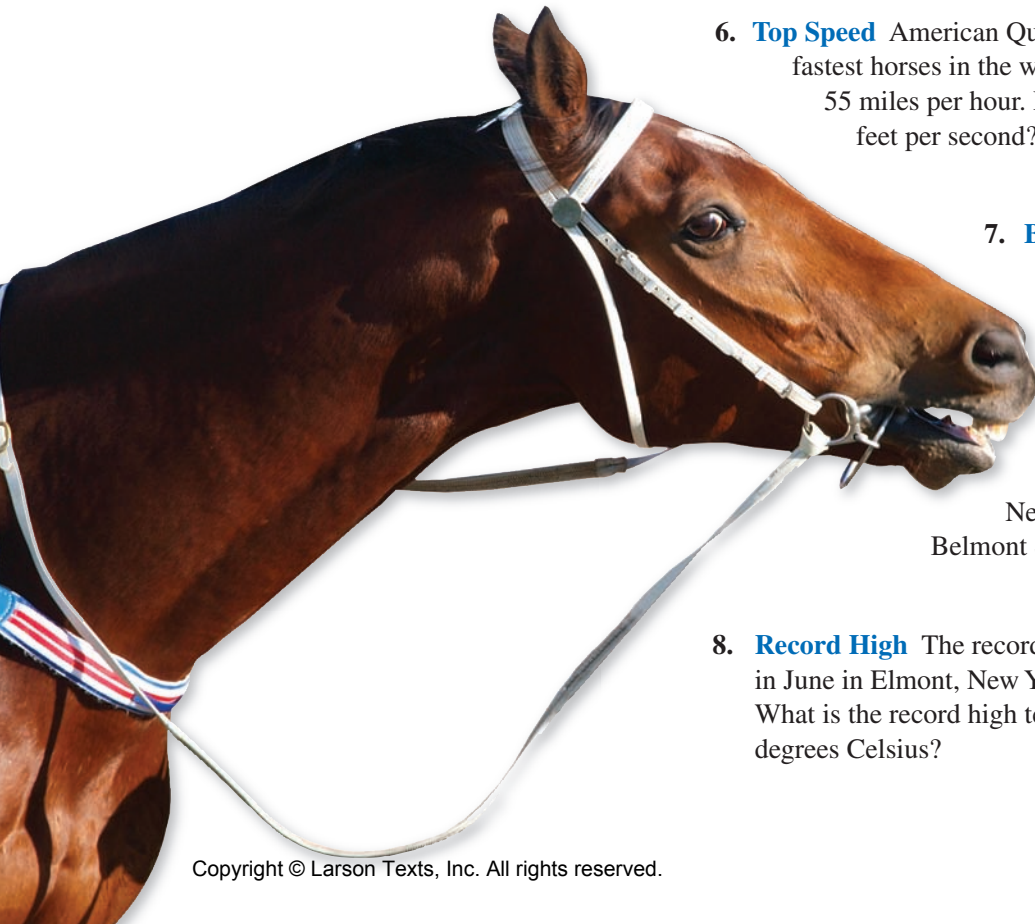


- Horse Height** The height of a horse is measured from the ground to its withers (shoulders). The traditional unit of measure is “hands,” where 1 hand is equal to 4 inches. A horse is 15 hands high. What is its height in inches?

- Top Speed** American Quarter Horses are the fastest horses in the world. They can run 55 miles per hour. How fast is this in feet per second?

- Belmont Stakes** The length of a horse race is often given in furlongs, where 1 furlong is equal to 0.125 mile. The Belmont Stakes is a 1.5-mile horse race that takes place every June at Belmont Park in Elmont, New York. How long is the Belmont Stakes in furlongs?

- Record High** The record high temperature in June in Elmont, New York, is 101°F. What is the record high temperature in degrees Celsius?



Chapter 1 Summary

Section Objectives

How does it apply to you?

Section 1

Use the order of operations to evaluate a numerical expression.



You need to know the order of operations to do calculations properly in your daily life. (See Examples 1 and 2.)

Use a calculator to evaluate a numerical expression.



To get an accurate answer, you must use the correct keystroke sequence when using a calculator. (See Example 3.)

Use the order of operations to evaluate a formula.



You will use common formulas many times throughout your life. (See Examples 4, 5, and 6.)

Section 2

Round numbers in a real-life context.



Using “exact” numbers is often unreasonable. It is usually reasonable to round numbers according to the context. (See Examples 1 and 2.)

Read large and small numbers.



Large and small numbers are often written in *scientific* or *exponential notation*. (See Examples 3 and 4.)

Understand the concept of “garbage in, garbage out.”



When answering a question, remember that the accuracy of the output is only as good as the accuracy of the input. (See Examples 5 and 6.)

Section 3

Understand and find a percent of a number.



Percents are everywhere. You need to have a firm grasp of percents to make sense of today’s world. (See Examples 1 and 2.)

Determine what percent one number is of another number.



Percent can be used to compare two numbers in the same context. (See Examples 3 and 4.)

Use percent to represent change.



Percent can be used to describe the amount that a quantity changes over time. (See Examples 5 and 6.)

Section 4

Use unit analysis to “balance” both sides of a formula.



Everything you measure has a number with some sort of unit of measure attached. (See Examples 1 and 2.)

Convert within a given system of measure.



You need to know how to compare units. (See Examples 3 and 4.)

Convert between different systems of measure.



Two systems of measure are used in the United States. You need to know how to compare units. (See Examples 5 and 6.)

Chapter 1 Review Exercises

Section 1.1

1. **Train Fare** A family of six takes a one-way trip on a train. The cost is \$60 per adult and \$45 per child. Which of the following keystroke sequences is better for finding the total cost? Explain your reasoning.

a. $(2 \times 60) + (4 \times 45) =$

b. $2 \times (60 + 4 \times 45) =$

2. **Seating** The seating area of a passenger car on a high-speed train is in the shape of a rectangle. It has a length of 70 feet and a width of 9 feet. Estimate how many seats can fit in the passenger car. Explain your reasoning.



High-speed Rail In Exercises 3–6, use the facts about high-speed rail in China.



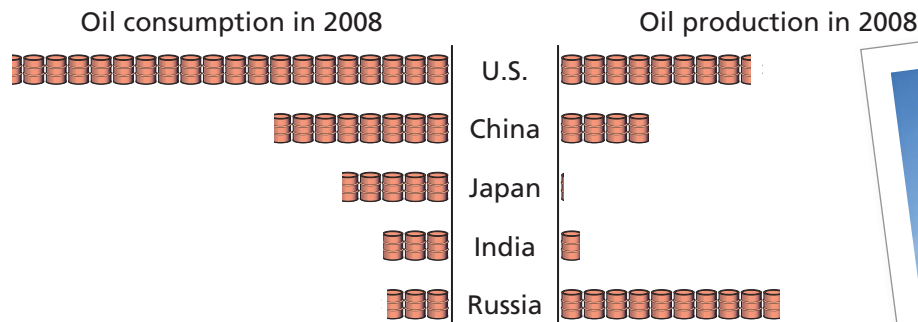
- In 2010, a Chinese passenger train hit a record speed of 302 miles per hour during a test run.
- It reached the top speed on a segment of the 824-mile-long line.
- The line is expected to open in 2012 and will halve the current travel time between Beijing and Shanghai to five hours.
- China already has the world's longest high-speed rail network, and it plans to cover 8125 miles by 2012 and 10,000 miles by 2020.

3. At its top speed, about how long does it take the passenger train to travel 1050 miles?
4. How long did it take to travel between Beijing and Shanghai before 2012?
5. From 2012 to 2020, about how many miles per year does China plan to expand its high-speed rail network?
6. The track distance between Beijing and Shanghai is 819 miles. Why do you think it takes five hours to make this trip?

Section 1.2

Oil In Exercises 7–10, use the pictograph.

Top Five Oil-consuming Countries in 2008

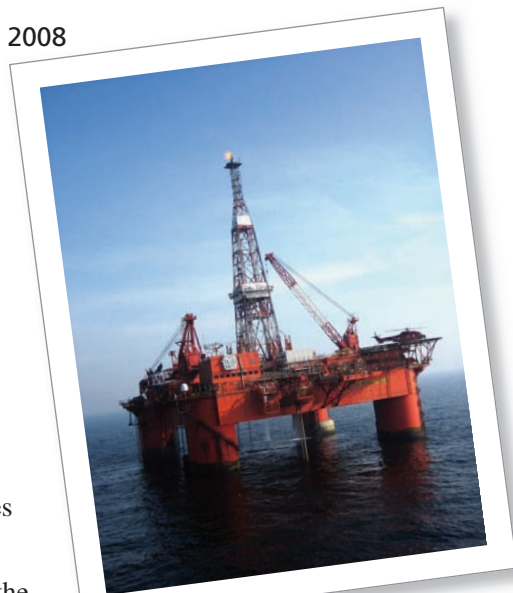


Key: = 1,000,000 barrels of oil per day

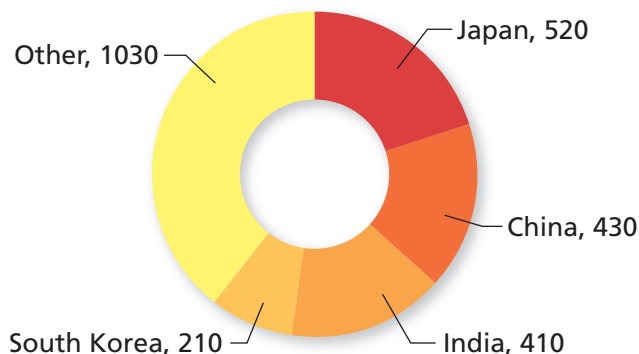
7. Consider the consumption side of the pictograph.
 - a. Estimate the number of barrels of oil consumed by the United States per day. Write your answer in standard decimal notation.
 - b. For how many years do you think the United States and the rest of the world can continue consuming oil at this rate? Explain your reasoning.
8. Estimate the number of barrels of oil produced by Russia per day. Write your answer in exponential notation.
9. A barrel of oil contains 42 gallons of oil.
 - a. In 2008, the population of China was about 1.3 billion. Estimate the number of gallons of oil consumed by China per person per day.
 - b. In 2008, the population of the United States was about 304 million. Compare your estimate in part (a) with the number of gallons of oil consumed by the United States per person per day.
10. Use the formula below to estimate and compare the net exports for Russia and Japan. What does a negative value indicate?

$$\text{Net exports} = \text{oil produced} - \text{oil consumed}$$

11. **Iran** Use the information in the doughnut graph to estimate the total number of barrels of oil Japan imported from Iran in 2008. Looking at the pictograph above, why do you think that Japan is one of Iran's top importers?
12. **U.S. Sanctions** The United States does not import any oil from Iran because of economic sanctions against the Middle Eastern country. It is estimated that lifting the sanctions could save the United States \$76 billion per year. Suppose sanctions were lifted. Estimate how much the United States would save per day.

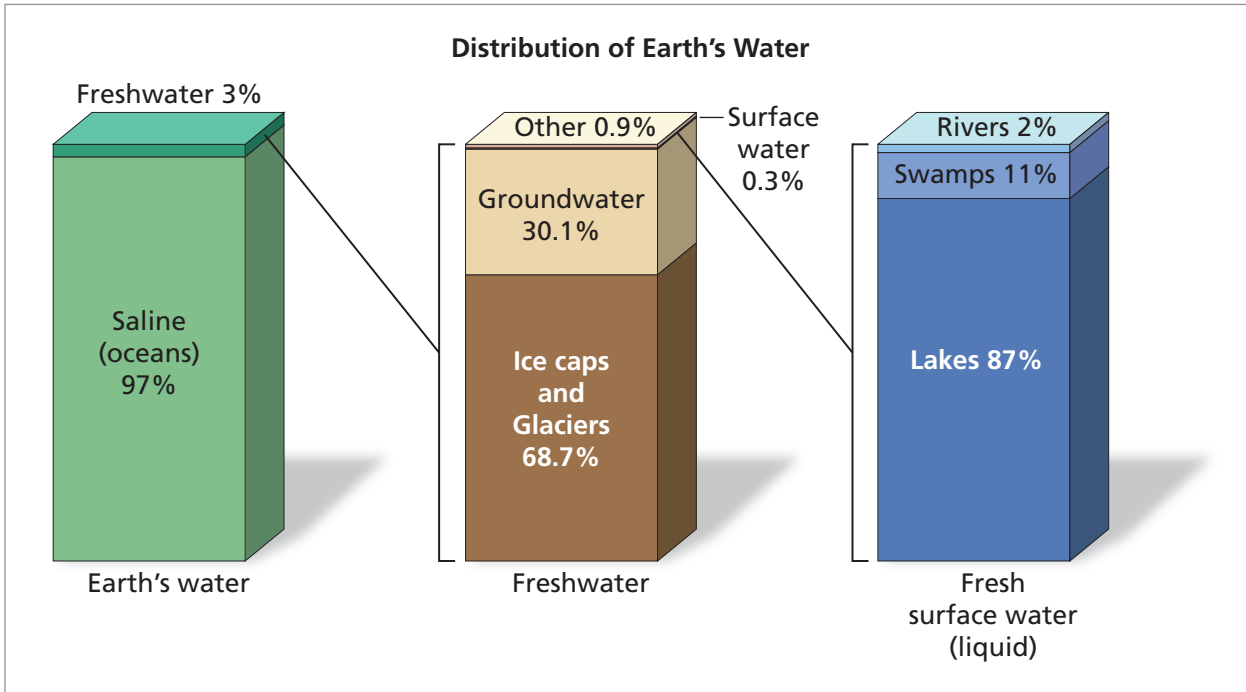


Destination of Oil Exports from Iran in 2008
(in thousands of barrels of oil per day)



Section 1.3

Earth's Water Distribution In Exercises 13–20, use the graph.



13. There are about 332.5 million cubic miles of water on Earth. How many cubic miles of Earth's water are freshwater?
14. How much freshwater is frozen in ice caps and glaciers?
15. The lakes on Earth hold 42,320 cubic miles of water. Of this, 20,490 cubic miles are saline. The rest are freshwater. What percent of the water in lakes is freshwater?
16. Groundwater accounts for about 5.6 million cubic miles of water. What percent of Earth's water is underground?
17. Suppose climate change causes some of the ice caps and glaciers to melt, adding 6000 cubic miles of freshwater to the surface water. Use percent to describe the increase in surface water.
18. What effect would a significant increase in surface water have on Earth and its inhabitants?
19. One of the categories in the graph is "Other." Where else do you think you can find freshwater besides the categories shown?
20. How do you think scientists estimated that there are about 332.5 million cubic miles of water on Earth?



Section 1.4

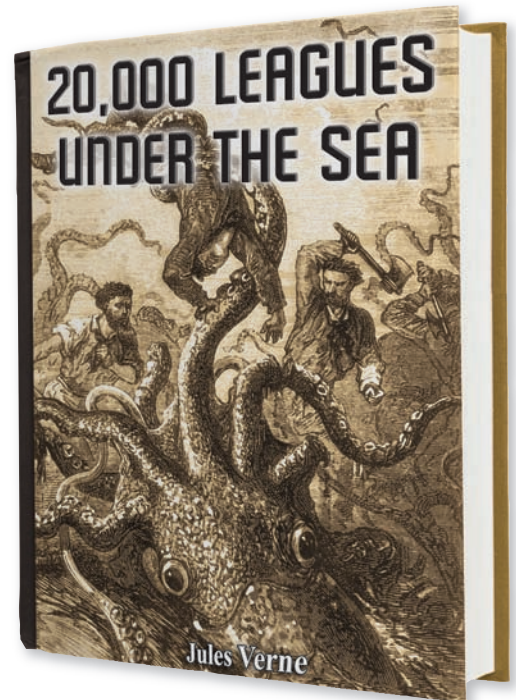
Nautical Units In Exercises 21–26, use the table.

21. During the U.S. Civil War, the Confederate ironclad CSS *Virginia* clashed with the Union ironclad USS *Monitor* in the Battle of Hampton Roads. The speed of the *Monitor* was about 9 miles per hour, and the speed of the *Virginia* was about 6 knots. Which ship was faster?
22. The phrase “deep six” means to throw something away. It was originally used by boaters to mean 6 fathoms. Convert this depth to feet.
23. Territorial waters around the United States used to be defined by the reach of a cannonball fired from shore, or a “cannon shot.” One cannon shot was defined as 3 nautical miles.
 - a. Convert 1 cannon shot to miles.
 - b. Compare this definition of territorial waters to the modern definition.
24. Jules Verne wrote a famous novel called *20,000 Leagues Under the Sea*.
 - a. Convert 20,000 leagues to miles.
 - b. At its deepest point, the ocean is about 7 miles deep. What do you think the title of Verne’s novel means? Explain your reasoning.
25. Below is an excerpt from William Shakespeare’s *The Tempest*.

*Full fathom five thy father lies;
Of his bones are coral made;
Those are pearls that were his eyes;
Nothing of him that doth fade
But doth suffer a sea-change
Into something rich and strange.*

Shakespeare writes that the man lies 5 fathoms deep. Convert this depth to feet.

1 fathom = 2 yards
1 nautical mile \approx 1013 fathoms
1 nautical mile \approx 1.15 miles
1 league = 3 nautical miles
1 league \approx 3.45 miles
1 knot = 1 nautical mile/hr



26. Mark Twain was the pen name of Samuel Clemens, who spent several years as a steamboat pilot. He chose the name because the leadsman on a steamboat would call out “mark twain” to indicate that the water depth was 12 feet and safe for the boat to pass. Convert this depth to fathoms.

2

The Mathematics of Consumption

2.1 Unit Prices

- ▶ Find the unit price of an item.
- ▶ Compare the unit prices of two or more items.
- ▶ Find the annual cost of an item.

2.2 Markup & Discount

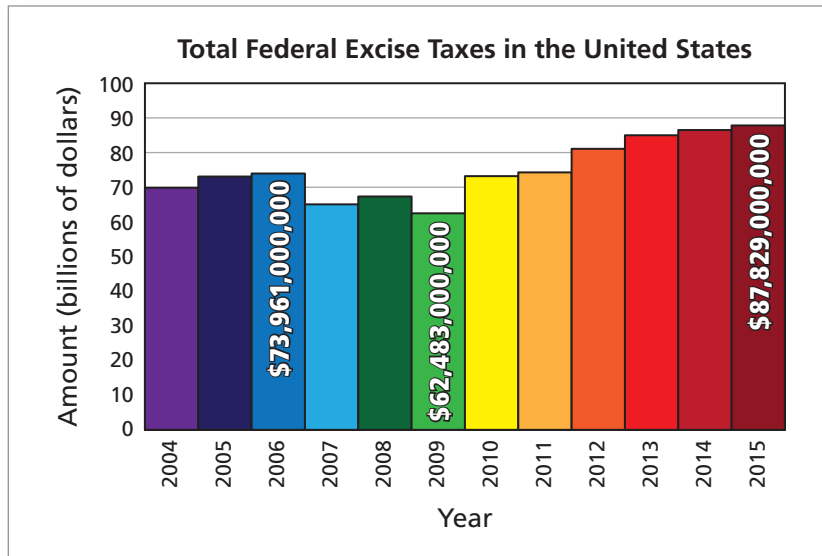
- ▶ Find the markup on an item.
- ▶ Find the discount on an item.
- ▶ Find the final price after multiple discounts.

2.3 Consumption Taxes

- ▶ Find the sales tax on an item.
- ▶ Find the excise tax on an item.
- ▶ Find the value-added tax on an item.

2.4 Budgeting

- ▶ Create and balance a monthly budget.
- ▶ Write checks and balance a checkbook.
- ▶ Analyze a budget.



Example 4 on page 77 describes the changes in federal excise tax revenues. Why did the excise tax revenue decline from 2006 to 2009?



2.1 Unit Prices

- ▶ Find the unit price of an item.
- ▶ Compare the unit prices of two or more items.
- ▶ Find the annual cost of an item.

Unit Prices

The **unit price** of an item is the cost per pound, quart, or other unit of weight or volume. It is often posted on the shelf tag below the item, along with the item price.



	\$2.75	Item price
* North Coast Co-op * ARROWHEAD MILLS CORN GRITS - Organic Yellow 0743338525 00179 12	24 OZ	Total units
	0.11/OZ	Unit price

Finding a Unit Price

To find a unit price, divide the item price by the total number of units.

$$\text{Unit price} = \frac{\text{item price}}{\text{total units}}$$

EXAMPLE 1 Finding a Unit Price

Check that the unit price shown above is correct.

SOLUTION

$$\begin{aligned} \text{Unit price} &= \frac{\text{item price}}{\text{total units}} \\ &= \frac{\$2.75}{24 \text{ oz}} \approx \$0.1146 \end{aligned}$$

Rounded to the nearest cent, the unit price is \$0.11 per ounce.

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Find the unit price of each item.



\$853.65 for 8 oz



\$119 for 1.7 fl oz



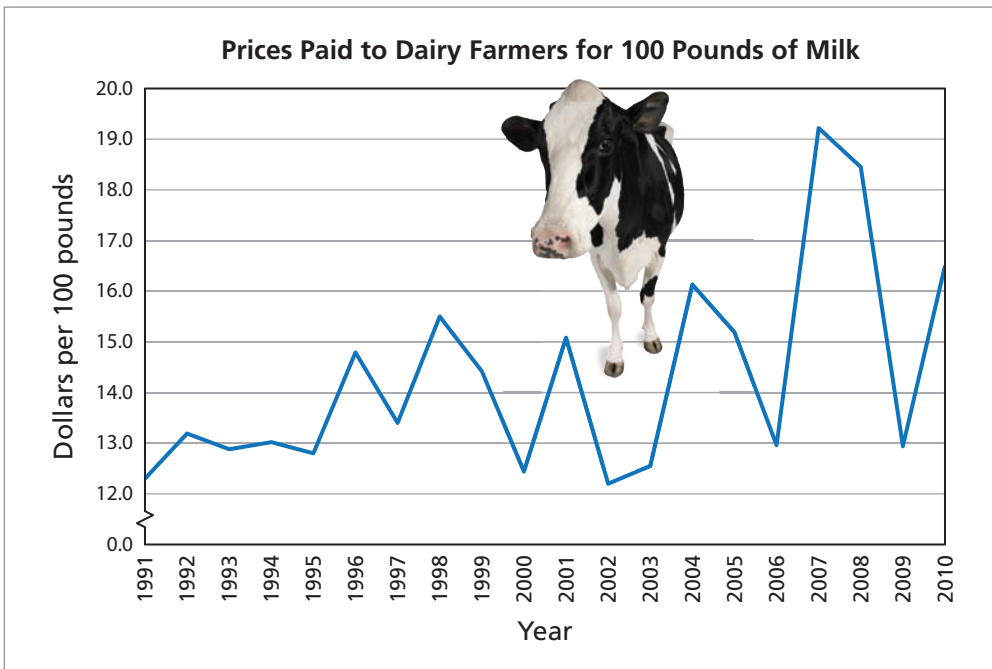
\$10.50 for 8 oz

EXAMPLE 2 Analyzing a Unit Price

Dairy farmers are paid for every 100 pounds of milk produced.

- In 2010, how much did a dairy farmer earn per gallon of milk produced? (A gallon of whole milk weighs about 8.6 pounds.)
- In 2010, the price of a gallon of milk was about \$3.25. What percent of the price did the dairy farmer earn?

Study Tip
 An old saying is "A pint is a pound the world around." The actual weight of a pint of water is about 1.04 pounds. There are 8 pints in a gallon, so a gallon of water weighs about 8.32 pounds.



SOLUTION

- In 2010, a dairy farmer earned about \$16.50 for every 100 pounds of milk produced.

$$\text{Unit price} = \frac{\$16.50}{100 \text{ lb}} = \frac{\$16.50}{100} \left(\frac{8.6 \cancel{\text{ lb}}}{1 \text{ gal}} \right) = \frac{\$1.419}{\text{gal}}$$

A dairy farmer earned about \$1.42 per gallon of milk produced.

- $$\frac{\text{Amount earned by farmer}}{\text{Price per gallon}} = \frac{\$1.42}{\$3.25} \approx 0.44$$

The dairy farmer earned about 44% of the price.

For milk to be labeled "organic," the cows must meet four criteria.

- Not treated with growth hormone
- Not given antibiotics while in a herd
- Not fed feed treated with pesticides
- Must have access to pasture

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In 2010, a dairy farmer earned about \$25 for every 100 pounds of milk produced organically. The price of a gallon of organic milk was about \$5.25.

- How much did a dairy farmer earn for a gallon of organic milk in 2010?
- What percent of the price did the dairy farmer earn?

Comparing Unit Prices

EXAMPLE 3 Comparing Unit Prices

Compare the unit prices of the three laundry detergents.

a. Brand A



b. Brand B



c. Brand C



Study Tip

To compare unit prices, you need the same units. That is why a conversion factor is used to calculate the unit price of brand B.

SOLUTION

$$\text{a. Unit price} = \frac{\text{item price}}{\text{total units}} = \frac{\$12.99}{100 \text{ fl oz}} \approx \$0.13 \text{ per fl oz}$$

$$\text{b. Unit price} = \frac{\text{item price}}{\text{total units}} = \frac{\$17.99}{2 \cancel{\text{ gal}} \left(\frac{1 \cancel{\text{ gal}}}{128 \text{ fl oz}} \right)} \approx \$0.07 \text{ per fl oz}$$

$$\text{c. Unit price} = \frac{\text{item price}}{\text{total units}} = \frac{\$7.99}{50 \text{ fl oz}} \approx \$0.16 \text{ per fl oz}$$

Brands A and C are comparable, with brand A being a little less per fluid ounce. Brand B has a considerably lower unit price.

✓ Checkpoint

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Each of the above detergents recommends using 2 fluid ounces per load. Compare the cost per load of brand B with the cost per load of homemade laundry soap.

Homemade Laundry Soap (Use 1/2 cup per load)

- * 1/3 bar Fels-Naptha Soap (\$0.40)
- * 1/2 cup washing soda (\$0.17)
- * 1/2 cup borax powder (\$0.14)

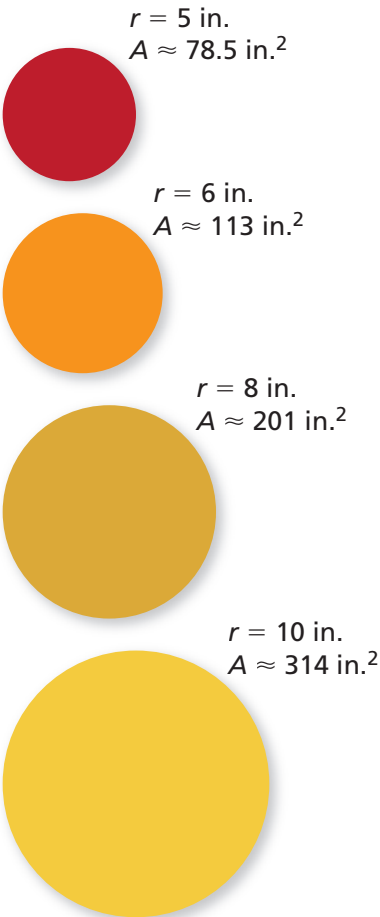
Preparation: Grate the soap and put it in a cooking pan. Add 6 cups water and heat until the soap melts. Add the washing soda and borax and stir until dissolved. Pour 4 cups hot water into a bucket. Add the soap mixture and stir. Then add 1 gallon plus 6 cups water and stir. Let the soap mixture stand for about 24 hours and it will gel.



EXAMPLE 4 Comparing Unit Prices

Compare the unit prices of the different sizes of pizza.

- Personal (10-inch diameter):
\$5.99
- Small (12-inch diameter):
\$9.99
- Medium (16-inch diameter):
\$13.99
- Large (20-inch diameter):
\$18.99



Notice that when the radius of the pizza doubles, the area is four times greater.

SOLUTION

Use the formula for the area of a circle to find the area of each pizza. Remember that the formula for the area of a circle is $A = \pi r^2$, where π is approximately equal to 3.14 and the radius r is half the diameter.

- Area $\approx 3.14(5^2) = 78.5$ in.²
 Unit price $\approx \frac{\$5.99}{78.5 \text{ in.}^2} \approx \0.076 per sq in. **Personal**
- Area $\approx 3.14(6^2) \approx 113$ in.²
 Unit price $\approx \frac{\$9.99}{113 \text{ in.}^2} \approx \0.088 per sq in. **Small**
- Area $\approx 3.14(8^2) \approx 201$ in.²
 Unit price $\approx \frac{\$13.99}{201 \text{ in.}^2} \approx \0.070 per sq in. **Medium**
- Area $\approx 3.14(10^2) = 314$ in.²
 Unit price $\approx \frac{\$18.99}{314 \text{ in.}^2} \approx \0.060 per sq in. **Large**

The small pizza has the greatest unit price. The large pizza has the least unit price.

✓ Checkpoint

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Find the unit price of a jumbo pizza.

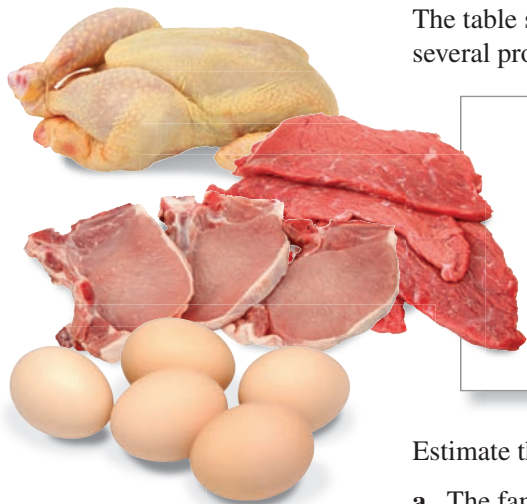
- Jumbo (24-inch diameter): \$24.99

Using Unit Prices to Find Annual Savings

Because unit prices often differ by small amounts, people often ignore the differences and think, “It’s just a few cents different.” However, you can get a better idea of the differences when you compare them on the basis of annual consumption.

EXAMPLE 5 Finding the Annual Savings

The table shows the average annual consumption by Americans for several products.



Average Annual Consumption by Americans

Beef	61.2 lb	Chicken	58.8 lb	Pork	46.0 lb
Turkey	13.9 lb	Margarine	4.2 lb	Eggs	247
Whole milk	6.1 gal	Tea	8.0 gal	Butter	5.0 lb
Cheese	32.4 lb	Wheat flour	136.6 lb	Sugar	65.7 lb

Estimate the annual savings for a family of four people for the following.

- The family purchases all of its whole milk at a discount store in which the whole milk averages \$1.20 less per gallon than at a supermarket.
- The family purchases all of its meat at a discount store in which the meat averages \$0.85 less per pound than at a supermarket.

SOLUTION

- The total amount of whole milk consumed (per person) is 6.1 gallons.

For a family of 4, this amounts to

$$4(6.1) = 24.4 \text{ gallons.}$$

If the family was able to save \$1.20 per gallon, the annual savings would be

$$1.20(24.4) = \$29.28.$$

- The total amount of meat consumed (per person) is

$$61.2 + 58.8 + 46.0 + 13.9 = 179.9 \text{ pounds.}$$

For a family of 4, this amounts to

$$4(179.9) = 719.6 \text{ pounds.}$$

If the family was able to save \$0.85 per pound, the annual savings would be

$$0.85(719.6) = \$611.66.$$

✓ Checkpoint

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Estimate the annual savings for a family of 4 people when the family purchases all of its wheat flour and sugar at a discount store at an average savings of \$0.67 per pound.

EXAMPLE 6 Eating In or Eating Out?

The list shows the ingredients for six cheeseburgers.

\$2.84	1 1/2 lb hamburger
\$2.99	6 hamburger buns
\$0.90	2 tomatoes
\$0.35	1 onion
\$2.00	6 cheese slices
\$0.59	1 head lettuce



- Estimate the cost per cheeseburger.
- You can buy a quarter-pound cheeseburger for \$3.49. How much will you save by making six cheeseburgers instead of buying six?
- Suppose that once a week you make six cheeseburgers instead of buying six. How much will you save in a year?

SOLUTION

- a. The cost of the ingredients is

$$2.84 + 2.99 + 0.90 + 0.35 + 2.00 + 0.59 = \$9.67.$$

The unit price per cheeseburger is

$$\text{Unit price} = \frac{\$9.67}{6 \text{ cheeseburgers}} \approx \$1.61.$$

- b. The cost of buying 6 cheeseburgers is

$$6(3.49) = \$20.94.$$

You will save $20.94 - 9.67 = \$11.27$.

- c. By saving the amount in part (b) once a week for a year, you will save

$$(11.27)(52) = \$586.04.$$

✓ Checkpoint

You are preparing spaghetti for four people.

\$1.59	1 box spaghetti
\$1.89	1 lb hamburger
\$3.49	1 jar spaghetti sauce
\$2.89	1 package parmesan cheese
\$0.35	1 onion
\$0.79	1 can sliced mushrooms

- What is the unit price per person?
- How much more would you pay to buy 4 spaghetti meals at a restaurant for \$7.95 each?

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2.1 Exercises

Groceries In Exercises 1–4, find the unit price of the item. (See Example 1.)

1. Peanuts



\$2.79 for 16 oz

2. Ramen



\$4.29 for 72 oz

3. Grape juice



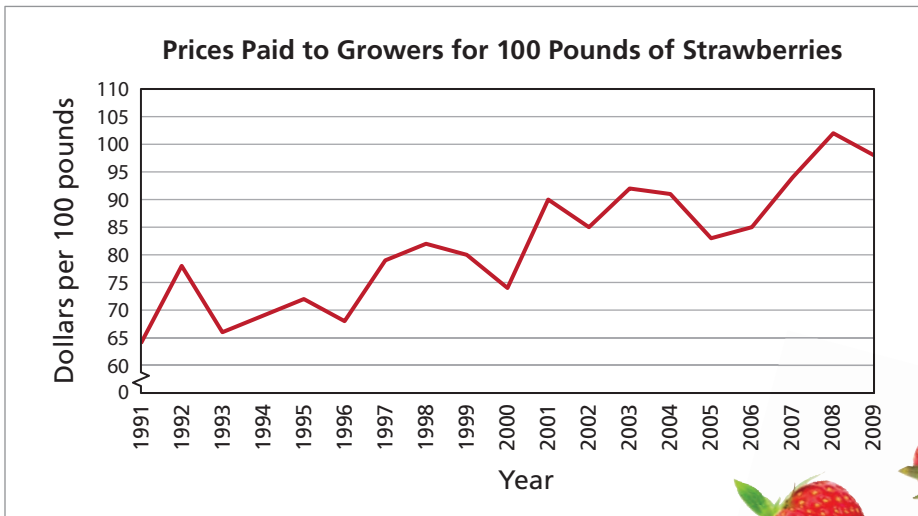
\$3.39 for 64 fl oz

4. Ice cream



\$4.49 for 1.5 qt

Strawberries In Exercises 5 and 6, use the graph. (See Example 2.)



5. Strawberry growers are paid for every 100 pounds of strawberries produced.

- a. In 2009, how much did a grower earn for a pound of strawberries?
- b. In 2009, the price of a 16-ounce container of strawberries was \$2.57. What percent of the price did the grower earn?

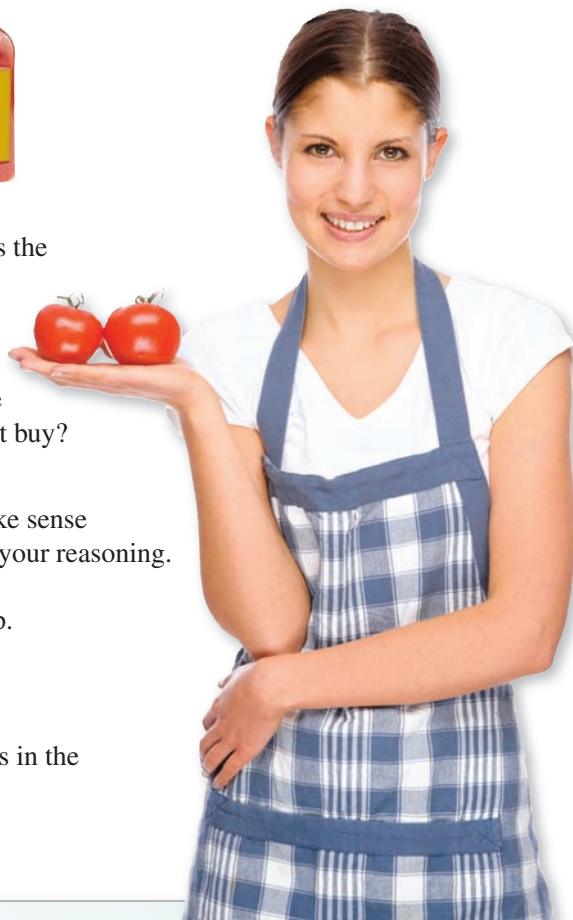
6. In 1991, the average retail price of a pound of strawberries was \$1.32.

- a. In 1991, how much did a grower earn for a pound of strawberries?
- b. In 1991, what percent of the average retail price did the grower earn?
- c. Compare the percent in part (b) with the percent from Exercise 5. Did growers earn a larger portion of the average retail price in 1991 or in 2009? Explain your reasoning.

Ketchup In Exercises 7–10, use the information below. (See Examples 3 and 4.)



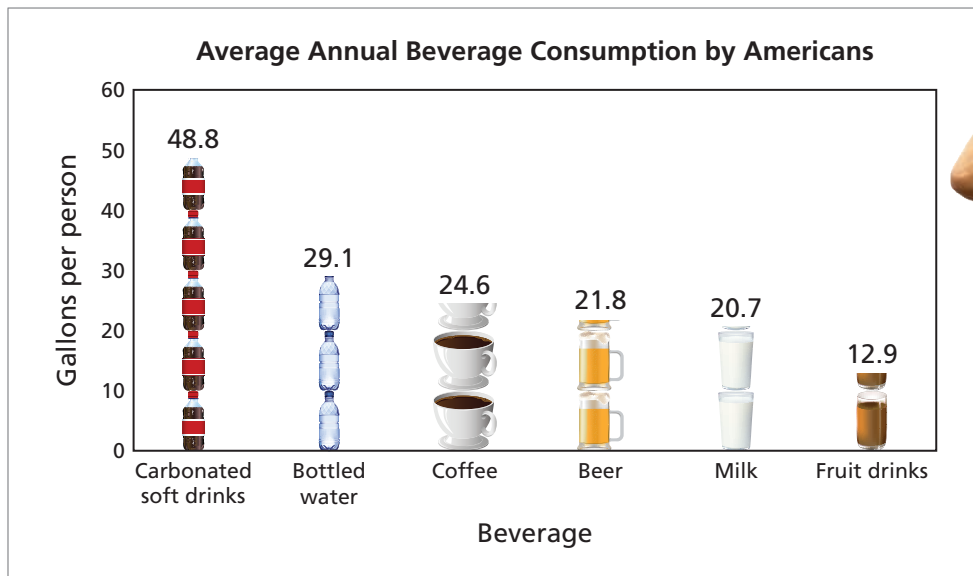
7. Compare the unit prices of the three brands of ketchup. Which is the best buy? Explain your reasoning.
8. Brand D contains 114 ounces and costs \$8.38.
 - a. Compare the unit price of brand D with the unit prices of the three brands above. Is the largest product necessarily the best buy? Explain your reasoning.
 - b. You use less than 20 ounces of ketchup per year. Does it make sense for you to buy the brand with the lowest unit price? Explain your reasoning.
9. A 40-ounce bottle of ketchup contains about 4.2 cups of ketchup. (One cup equals 8 fluid ounces.)
 - a. What is the unit price of brand C in dollars per fluid ounce?
 - b. Why do you think a manufacturer would prefer to use ounces in the unit rate?
10. Below is a recipe for homemade ketchup.



Homemade Ketchup (Makes 48 ounces)			
* 4 lb tomatoes	(\$7.99)	* 1½ onions	(\$0.37)
* 1 tsp ground cloves	(\$0.65)	* 1 tsp salt	(\$0.01)
* 1 tsp allspice	(\$0.28)	* 1 cup vinegar	(\$0.24)
<p>Preparation: Place the tomatoes in a boiling pot of water and leave them in until the skins split. Strain the tomatoes and allow them to cool. Peel the skins, slice the tomatoes open, and remove the seeds. Dice the tomatoes and onions and put them in a saucepan. Simmer for 10 minutes. Pour into a blender. After blending, pour back into saucepan and add the other ingredients. Stir the mixture. Simmer for 1½ hours. Stir often. After one-third has burned off, turn heat off. Allow time to cool.</p>			

- a. Compare the unit price of homemade ketchup with the unit price of brand A above.
- b. Your neighbor gives you 4 pounds of tomatoes from his garden. What is the unit price of homemade ketchup when you do not have to pay for the tomatoes?

Beverage Consumption In Exercises 11–14, use the graph. (See Examples 5 and 6.)



Some brands of bottled water indicate on the label that the water is from a public water source. This means that the bottle contains purified tap water.

11. At a discount store, fruit drinks average \$1.08 less per gallon than at a supermarket. A family of four people purchases all of its fruit drinks at the discount store. Estimate the annual savings.
12. At a discount store, ground coffee averages \$0.10 less per ounce than at a supermarket. Four college roommates purchase all their ground coffee at the discount store. Estimate the annual savings. (7.68 ounces of ground coffee make 1 gallon of coffee.)
13. At a supermarket, a 24-pack of 16.9-fluid-ounce bottles of water costs \$4.99.
 - a. At a discount store, a 35-pack of 16.9-fluid-ounce bottles of water costs \$4.49. Suppose you buy all your bottled water at the discount store instead of at the supermarket. How much will you save per year?
 - b. Your tap water rate is \$2.62 per 1000 gallons of water. Suppose you use tap water instead of buying bottled water at the supermarket. How much will you save per year?
14. You use 0.5 fluid ounce of coffee creamer with each 6-fluid-ounce “cup” of coffee. The creamer that you use costs \$2.19 for 16 fluid ounces. You can make homemade coffee creamer using the recipe shown.

Homemade Coffee Creamer (Makes 22 fluid ounces)

* 1 14-oz can sweetened condensed milk	(\$1.49)
* 1½ cups milk	(\$0.29)
* 1 tbsp vanilla extract	(\$0.70)

Preparation: Pour the ingredients into a jar and shake well.

- a. Estimate how much you could save annually by making your own creamer.
- b. The vanilla extract is optional. Estimate how much you could save annually by making your own creamer without the vanilla instead of buying the creamer at the store.

▶ Extending Concepts

Product Downsizing Instead of increasing the price of a product, some manufacturers decrease the amount of the product in a package but charge the same price. In Exercises 15–18, analyze the effect of product downsizing on unit price.

15. What is the difference in unit price?



Old size: 7 oz
Price: \$0.89



New size: 5 oz
Price: \$0.89

16. What is the difference in unit price?



Old size: 64 fl oz
Price: \$2.59



New size: 52 fl oz
Price: \$2.59

17. A manufacturer decreases the amount of product in a package but keeps the price the same. What effect does this have on the unit price of the product? Explain your reasoning.

18. Is it possible to increase the unit price of a product by decreasing both the size and the price? Explain your reasoning.

Paying More for Less The weight or volume listed on a product may be greater than the actual amount of the product that you are receiving. In Exercises 19 and 20, analyze how you could end up paying more money for less product.

19. A 1-pound package of chicken costs \$1.99.

- The label on the package says “up to 15% solution.” This means that 15% of the weight of the package is a solution. What is the actual unit price per pound of chicken?
- Compare the unit price in part (a) to the unit price on the package.

20. Two brands of bleach and their prices are shown.



Brand A
96 fl oz
\$1.79



Brand B
182 fl oz
\$2.12

- Find the unit prices of the two brands in dollars per gallon. Based on your calculations, which brand is the better buy?
- Brand A is 6% bleach. Brand B is 3% bleach. You can make 3% bleach by mixing 1 fluid ounce of brand A with 1 fluid ounce of water. How many fluid ounces of 3% bleach can you make using brand A?
- Find the unit prices of the two brands in dollars per gallon of 3% bleach. Based on your calculations, which brand is the better buy?



2.2 Markup & Discount

- ▶ Find the markup on an item.
- ▶ Find the discount on an item.
- ▶ Find the final price after multiple discounts.

Finding the Markup on an Item

Markup is the difference between the retail price and the price the retailer pays for the item, or wholesale price.

Finding a Markup

The markup on an item is the difference between the retail price and the wholesale price.

$$\text{Markup} = \text{retail price} - \text{wholesale price}$$

To find the **markup percent**, divide the markup by the wholesale price.

$$\text{Markup percent} = \frac{\text{markup}}{\text{wholesale price}}$$

In fairness to retailers, you should remember that markup is the only way for a retailer to make a profit. Markup is not equal to profit. Out of the markup, a retailer has to pay for rent, utilities, taxes, salaries, benefits, and other business expenses.



EXAMPLE 1 Finding a Markup Percent

Markup percents vary greatly. Prescription drugs and jewelry tend to have high markup percents. Grocery items and everyday clothing tend to have low markup percents. Find the markup percent for each item.

- | | |
|---|--|
| <p>a. Pair of earrings
Wholesale price: \$89.00
Retail price: \$459.00</p> | <p>b. Pair of athletic socks
Wholesale price: \$1.13
Retail price: \$1.69</p> |
|---|--|

SOLUTION

$$\begin{aligned} \text{a. } \frac{459 - 89}{89} &= \frac{370}{89} \\ &\approx 4.157 \\ &\approx 416\% \end{aligned}$$

$$\begin{aligned} \text{b. } \frac{1.69 - 1.13}{1.13} &= \frac{0.56}{1.13} \\ &\approx 0.496 \\ &\approx 50\% \end{aligned}$$



✓ Checkpoint

Find the markup percent for each item.

- | | |
|--|---|
| <p>c. Automobile
Wholesale price: \$25,450
Retail price: \$27,990</p> | <p>d. Leather chair
Wholesale price: \$235
Retail price: \$799</p> |
|--|---|



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EXAMPLE 2 Finding a Markup Percent



You live near a designer handbag outlet store. The store sells discontinued and slightly damaged handbags at prices that are well below normal retail prices. You buy a handbag for \$195. The handbag normally retails for \$895. You then put the handbag on eBay® and sell it for \$395.

- What is your markup and markup percent?
- Is your markup the same as your profit? Explain.



Study Tip

“Wholesale price” is a relative term. In Example 2, you are the retailer, so the price you pay at the outlet store becomes your wholesale price.

SOLUTION

- Your markup is

$$\text{Markup} = 395 - 195 = \$200.$$

Your markup percent is

$$\begin{aligned} \text{Markup percent} &= \frac{\text{Markup}}{\text{Wholesale price}} \\ &= \frac{200}{195} \\ &\approx 1.026 \\ &= 102.6\%. \end{aligned}$$

- Your markup in this transaction is *not* the same as your profit. To find your profit, you must also subtract your expenses.

eBay® insertion fee	\$2.00	Fee charged for listing
Final value fee	\$35.55	9% of \$395
PayPal fee	\$11.76	2.9% of \$395 + \$0.30
Sales tax	\$11.70	6% paid at outlet store
Shipping	\$22.50	Mail handbag to customer
Other expenses	\$35.00	Supplies, transportation
Total	\$118.51	

Your profit is

$$\begin{aligned} \text{Markup} - \text{Expenses} &= 200.00 - 118.51 \\ &= \$81.49. \end{aligned}$$

So, your markup is not the same as your profit. If you spend 5 hours in traveling, shopping, Internet use, correspondence, wrapping, and shipping, then your hourly wage is \$16.30.

✓ Checkpoint

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A bookstore pays \$140 for a textbook and sells it for \$200. The bookstore manager determines that the retail price of the book is a 30% markup because \$60 is 30% of \$200. Is this a correct use of the term *markup*? Explain your reasoning.



Finding the Discount on an Item

Discount is the difference between the “regular” retail price of an item and the price that a consumer actually pays for the item.

Study Tip

Notice that for markup percent, the wholesale price is the base. However, for discount percent, the regular price is the base.

Finding a Discount

The discount on an item is the difference between the regular retail price and the discounted, or sale, price.

$$\text{Discount} = \text{regular price} - \text{discounted price}$$

To find the **discount percent**, divide the discount by the regular price.

$$\text{Discount percent} = \frac{\text{discount}}{\text{regular price}}$$

EXAMPLE 3 Using Discounts and Markup

You work in a clothing store. You order 25 T-shirts in assorted sizes. The wholesale price of each shirt is \$8.37. The regular retail price is \$24.99. During the next 2 months, you repeatedly mark down the price until you finally sell all 25 shirts. Find the average markup percent for the 25 shirts.

- 13 shirts at \$24.99
- 4 shirts at 25% off
- 6 shirts at 50% off
- 2 shirts at 75% off

SOLUTION

A spreadsheet works well to organize this problem.

	A	B	C	D	E	
1	Wholesale Price	Regular Price	Discount Percent	Discount Price	Quantity Sold	Revenue
2	\$8.37	\$24.99	0.0%	\$24.99	13	\$324.87
3	\$8.37	\$24.99	25.0%	\$18.74	4	\$74.96
4	\$8.37	\$24.99	50.0%	\$12.50	6	\$75.00
5	\$8.37	\$24.99	75.0%	\$6.25	2	\$12.50
6	Total					\$487.33
7						

You paid $25(8.37) = \$209.25$ for the shirts. Your total markup was $487.33 - 209.25 = \$278.08$. So, your average markup percent was

$$\frac{\text{Total markup}}{\text{Total wholesale price}} = \frac{278.08}{209.25} \approx 1.329 = 132.9\%$$

✓ Checkpoint

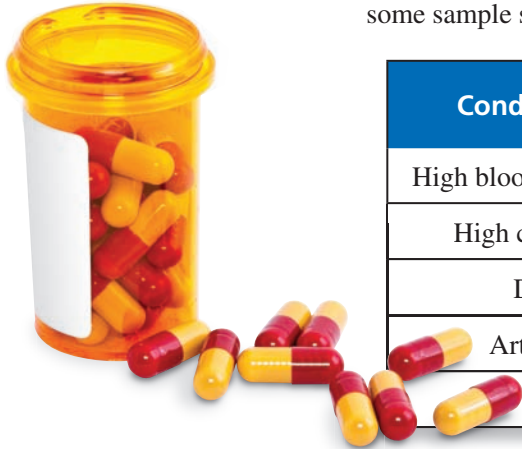
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Find the average markup percent for the following sales record.

- 11 shirts at \$24.99
- 7 shirts at 25% off
- 3 shirts at 50% off
- 4 shirts at 75% off

EXAMPLE 4 Buying Generic Prescription Drugs

In the United States, prescription drugs are often expensive. You can usually save money by asking your doctor to prescribe a generic version of a drug. Here are some sample savings.




Condition	Monthly Cost for Brand-Name Drug	Monthly Cost for Generic Substitute
High blood pressure	\$128	\$13
High cholesterol	\$95	\$37
Depression	\$103	\$37
Arthritis pain	\$135	\$30
Heartburn	\$179	\$24

Which of these represents the greatest discount percent?

SOLUTION

A spreadsheet works well to organize the information in this problem.



DATA

	A	B	C	D	E
1	Condition	Brand	Generic	Discount	Percent
2	High blood pressure	\$128.00	\$13.00	\$115.00	89.8%
3	High cholesterol	\$95.00	\$37.00	\$58.00	61.1%
4	Depression	\$103.00	\$37.00	\$66.00	64.1%
5	Arthritis pain	\$135.00	\$30.00	\$105.00	77.8%
6	Heartburn	\$179.00	\$24.00	\$155.00	86.6%
7					

Brand - Generic

Discount
Brand

The greatest discount percent is for the high blood pressure drug.

✓ Checkpoint

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Find the discount percents for the three drugs described in the article.

Crossing Borders for Prescription Drugs

Thousands of American seniors are going to Canada to stock up on something they can't seem to find at home—affordable prescription drugs.

Here are some examples. A stomach acid medication costs US\$129 in the US, but only US\$53 in Canada. An antihyperglycemic agent costs US\$52 in the US, but only US\$12 in Canada. Conjugated estrogens cost US\$26 in the US, but only US\$7 in Canada.

Finding the Final Price after Multiple Discounts

When you have two discounts on the same item, the final price can depend on the order in which you calculate the discounts.

EXAMPLE 5 Calculating Multiple Discounts

Each day in the United States, millions of coupons are distributed by mail, newspapers, and e-mails. You acquire a coupon for a pair of jeans.



A pair of jeans retails for \$40 and is being sold at a store that is having a “25% off” sale on all clothing.

- What is the final price when you first take 25% off, and then subtract \$10?
- What is the final price when you first subtract \$10, and then take 25% off?

SOLUTION

- Begin by taking 25% off.

$$\begin{array}{ccc} \boxed{\text{Original price}} & & \boxed{\text{25\% discount}} \\ & \searrow & \searrow \\ & 40 - 0.25(40) = 40 - 10 = \$30 \end{array}$$

Then use the coupon.

$$\begin{array}{c} \boxed{\text{Coupon}} \\ \searrow \\ 30 - 10 = \$20 \end{array}$$

The final price is \$20.

- Begin by using the coupon.

$$40 - 10 = \$30$$

Then take 25% off.

$$30 - 0.25(30) = 30 - 7.50 = \$22.50$$

The final price is \$22.50.

✓ Checkpoint

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Jeans were invented in 1873 by Levi Strauss and Jacob Davis.

Suppose the coupon in Example 5 is for 10% off, rather than \$10 off. Would the order in which you apply the discounts make a difference in the final price? Explain your reasoning.

EXAMPLE 6 Using Promotional Codes

A car rental company offers 10% off the cost of any rental car. The company also sends you an email with a promotional code for 20% off. You decide to use both discounts to reserve a car online.

Promotional Code:

The regular price for renting a car is \$70 per day. What is the final price after both discounts?

SOLUTION

Begin by taking 10% off.



$$\begin{array}{l} \text{Original price} \quad \text{10\% discount} \\ \downarrow \quad \downarrow \\ 70 - 0.10(70) = 70 - 7 \\ = \$63 \end{array}$$

Then take 20% off.

$$\begin{array}{l} \text{Discounted price} \quad \text{20\% discount} \\ \downarrow \quad \downarrow \\ 63 - 0.20(63) = 63 - 12.60 \\ = \$50.40 \end{array}$$

The final price is \$50.40.

✓ Checkpoint

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Use the information in the article to estimate a shopper's hourly wage for clipping grocery coupons from newspapers. Explain your reasoning.

Save Money by Using Coupons

In 2009, about 367 billion coupons were distributed in the United States. Of these, about 3.3 billion coupons were redeemed.

Studies have shown that shoppers who spend 20 minutes per week clipping coupons can save up to \$1000 per year. That is a 20% savings for a family with a \$5000 annual grocery bill.



2.2 Exercises

Apparel In Exercises 1–4, find the markup percent for the item. (See Examples 1 and 2.)



1.



Wholesale: \$18
Retail: \$40

2.



Wholesale: \$36
Retail: \$65.99

3.



Wholesale: \$12.18
Retail: \$19.99

4.



Wholesale: \$12.89
Retail: \$16.19





5. **Necklace** The wholesale price for a necklace is \$10. The markup percent is 200%. What is the markup?

6. **Watch** The wholesale price for a watch is \$25. The markup percent is 50%. What is the retail price?





Lawn and Garden You work in the lawn and garden section of a local retail store. You order 26 garden statues. The wholesale price of each statue is \$35. The regular retail price is \$70. During the next 4 months, you repeatedly mark down the price until you finally sell all 26 statues. In Exercises 7–10, use the sales record shown. (See Example 3.)

-  × 10 at \$70
-  × 8 at 25% off
-  × 5 at 50% off
-  × 3 at 75% off

7. Find the average markup percent for the 26 statues.
8. Is the revenue from the sale of the 26 garden statues equal to the profit? Explain your reasoning.
9. Your goal as a businessperson is to make a profit.
 - a. How many garden statues did you discount below the wholesale price?
 - b. How can you make a profit when you discount some of your inventory at less than the wholesale price? Explain your reasoning.
10. Create a spreadsheet similar to the one in Example 3 for the garden statue sales record. Experiment with the numbers in the “Quantity Sold” column. Remember that the total number of garden statues is 26.



	A	B	C	D	E	F
	Wholesale Price	Regular Price	Discount Percent	Discount Price	Quantity Sold	Revenue
1						
2	\$35.00	\$70.00	0.0%	\$70.00	10	\$700.00
3	\$35.00	\$70.00	25.0%	\$52.50	8	\$420.00
4	\$35.00	\$70.00	50.0%	\$35.00	5	\$175.00
5	\$35.00	\$70.00	75.0%	\$17.50	3	\$52.50
6	Total					\$1,347.50
7						

- a. Find four values such that the revenue is greater than the cost. (Cost = number of garden statues purchased × wholesale price)
- b. Find four values such that the revenue is less than the cost.
- c. Explain why you need to know this information to make a profit.

Cereal In Exercises 11 and 12, find the discount percent for the generic cereal. (See Example 4.)

11.



Generic
\$1.99

vs.



Brand Name
\$5.29

12.



Generic
\$2.39

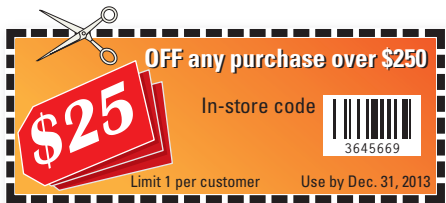
vs.



Brand Name
\$2.99

Television You are purchasing a television from an electronics store. The television retails for \$380, and the store is having a “15% off” sale on all items. In Exercises 13 and 14, use this information. (See Example 5.)

13. You have the coupon shown.



- What is the final price when you first take 15% off, and then subtract \$25?
 - What is the final price when you first subtract \$25, and then take 15% off?
14. You have the coupon shown.



- What is the final price when you first take 15% off, and then subtract \$20?
- What is the final price when you first subtract \$20, and then take 15% off?
- Suppose the coupon is for 20% off, rather than for \$20 off. What is the final price when you use the coupon with the sale?

Footwear A shoe store sends you a promotional code that you can use for a discount. In Exercises 15 and 16, use the information in the coupon. (See Example 6.)



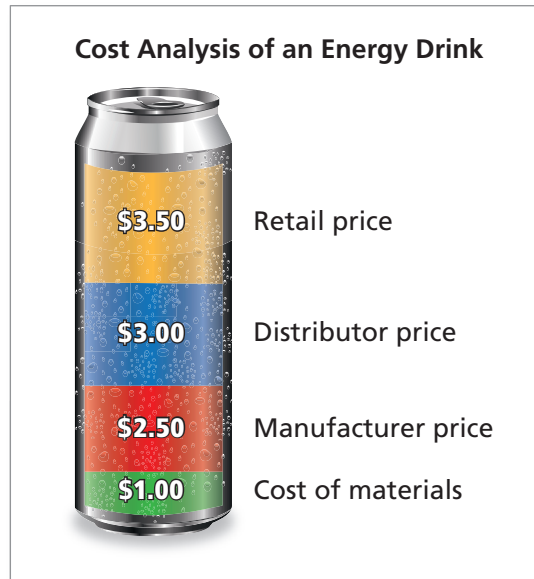
- What is the percent discount?
- The store is having a “60% off” sale on all items. What is the final price when you use the promotional code with the sale?



- What is the percent discount?
- The store is having a “50% off” sale on all items. What is the final price when you use the promotional code with the sale?

► Extending Concepts

Energy Drink In Exercises 17–20, use the display.



17. What is the markup percent from the manufacturer price to the retail price?
18. What is the markup percent from the distributor price to the retail price?
19. You buy one energy drink. You have a coupon for \$0.75 off. What is the discount percent?
20. Suppose the store marks up the energy drink to \$3.25 instead of \$3.50. What is the markup percent from the distributor price to the retail price?
21. **Clearance** A store advertises that it is having a “30% off” sale on all items, with an additional 20% off clearance items. The additional 20% is taken off after the 30% discount is applied. You are buying a spice rack that is on clearance. The regular price of the spice rack is \$30.
 - a. What is the total discount?
 - b. What is the total discount percent?
 - c. Why do you think the store advertises its sale using two discount percents instead of one? Explain.
22. **Customer Service** You work at a store that is having a “50% off” sale. A customer has a coupon for 50% off any item and thinks that a \$40 sweater should be free with the coupon. How would you explain why the sweater is not 100% off? Explain your reasoning.

2.1–2.2 Quiz

Liquid Hand Soap In Exercises 1–5, use the information below.

Brand A



7.5 fl oz
\$1.99

Brand B



19.95 fl oz
\$2.39

Brand C



32 fl oz
\$2.99

- Compare the unit prices of the three brands of liquid hand soap.
- A family of 4 uses 128 fluid ounces of liquid hand soap annually. Suppose the family purchases brand C instead of brand A. Estimate the annual savings for the family.
- The wholesale price of brand A is \$0.99.
 - Find the markup.
 - Find the markup percent.
- You have a coupon for 10% off brand B.
 - How much can you expect to pay for brand B?
 - How does the coupon affect the unit price of brand B? Explain your reasoning.
 - Does brand B give you the most hand soap for your money after you use the coupon? Explain your reasoning.
- Below is a recipe for homemade liquid hand soap.

Homemade Liquid Hand Soap

* 1 bar of soap	(\$ 0.33)	* 1 tsp glycerin	(\$ 0.14)
* 1 tbsp honey	(\$ 0.19)		

Grate the soap and put it in a blender with 1 cup of boiling water. Blend the soap and water. Then stir in the honey and glycerin. Let the mixture cool for 15 minutes, and then blend. Add cold water until you have 6 cups of mixture. Blend the mixture again and pour it into a container to let it cool. Do not put a lid on the container.

- What is the unit price of the homemade liquid hand soap?
- You make your own liquid hand soap instead of buying brand C. What is the discount percent?



Math & Currency

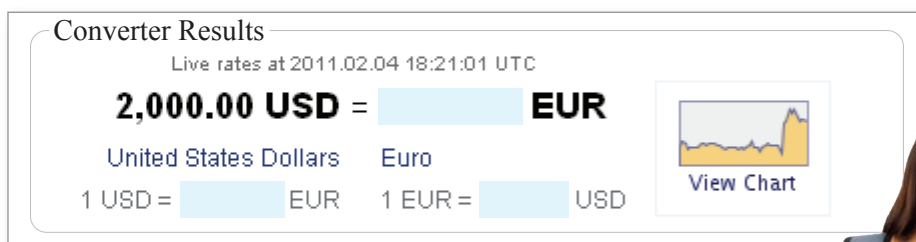
PROJECT: How Should I Exchange Currency?

- You are traveling from the United States to Europe and plan to take \$2000 in cash. You are trying to decide the best way to do this. To start, you use the *Universal Currency Converter*®* at *Math.andYou.com*. Using the converter, how many euros will you receive for \$2000 (USD)?

Universal Currency Converter

Convert: > From this currency:
 Enter an amount >
 Dutch (Netherlands) Guilders - NLG*
 Eastern Caribbean Dollars - XCD
 Egypt Pounds - EGP
 Estonia Krooni - EEK*
 Euro - EUR
[More currencies](#)

To this currency:
 United States Dollars - USD
 Euro - EUR
 United Kingdom Pounds - GBP
 Canada Dollars - CAD
 Australia Dollars - AUD
[More currencies](#) >



In Exercise 2, use the websites for the following to complete parts (a)–(f).

- local bank
 - airport currency exchange company
 - credit card company
 - foreign bank
- Which method of exchanging currency is the least costly?
 - Exchange your dollars for euros at your local bank.
 - Exchange your dollars for euros at the airport.
 - Use your credit card to make purchases in Europe.
 - Exchange your dollars for euros at a bank in the country you are visiting.
 - Buy traveler's checks at your local bank. Then exchange the traveler's checks for euros at a bank in the country you are visiting.
 - Use an ATM in the country you are visiting.
 - Using the methods in Exercise 2, will you receive as many euros as the *Universal Currency Converter*® indicates? If not, why?



*Provided by *XE.com*

2.3 Consumption Taxes

- ▶ Find the sales tax on an item.
- ▶ Find the excise tax on an item.
- ▶ Find the value-added tax on an item.

The Federalist No. 21

“It is a signal advantage of taxes on articles of consumption, that they contain in their own nature a security against excess. They prescribe their own limit; which cannot be exceeded without defeating the end proposed, that is, an extension of the revenue. . . . If duties are too high, they lessen the consumption; the collection is eluded; and the product to the treasury is not so great as when they are confined within proper and moderate bounds.”

Alexander Hamilton

Finding the Sales Tax on an Item

A **consumption tax** is a tax on spending. There are three basic types of consumption taxes: sales tax, excise tax, and value-added tax.

Consumption taxes do not tax savings, income, or property. Although personal and corporate income taxes provide most of the revenue to the federal government, consumption taxes are a primary source of income for state and local governments. The United States implemented consumption taxes much earlier than income taxes. The paragraph at the left was written by Alexander Hamilton in the Federalist Papers in 1787.

EXAMPLE 1 Finding Sales Tax

In the United States, sales tax rates vary greatly. Several states have no sales tax. There are states in which cities and counties have sales taxes, which are added on to state sales taxes. Find the sales tax on each item.

- a. Yacht: \$214,000
Sales tax: 8%



- b. Sports car: \$65,900
Sales tax: 7%



Study Tip

In Example 1a, you can estimate the sales tax to be “a little less than 10%.” Because 10% of \$214,000 is \$21,400, a sales tax of \$17,120 is reasonable.

SOLUTION

Retail price 8% tax rate

$$\text{a. } 214,000(0.08) = \$17,120$$
 The sales tax is \$17,120.

Retail price 7% tax rate

$$\text{b. } 65,900(0.07) = \$4613$$
 The sales tax is \$4613.

✓ Checkpoint

Find the sales tax on each item.

- c. High heel shoes: \$250
Sales tax: 6%



- d. Watch: \$1350
Sales tax: 7.4%

Help at Math.andYOU.com

Sales tax has become a quagmire of rules, misconceptions, tax forms, legislation, and legal battles. The basic problem is that the United States has thousands of sales tax jurisdictions (counting states, counties, and cities). Most jurisdictions interpret sales tax law to mean that a resident of the jurisdiction must pay sales tax on each item the resident purchases, regardless of where the item is purchased.

For instance, if you live in Washington State (which has sales tax) and purchase an item in Oregon (which has no sales tax), Washington expects you to report your purchase and pay sales tax. Even more common is the issue of Internet sales. If you run an Internet business, then you might have to collect and report sales tax when someone purchases an item from you.

EXAMPLE 2 Finding a Sales Tax Rate

Jacket	\$135.95
Subtotal:	\$135.95
Sales tax city:	\$ 1.36
Sales tax county:	\$ 4.35
Sales tax state:	\$ 9.25
TOTAL:	\$150.91
- THANKS FOR SHOPPING -	

You purchase a jacket in an airport mall. You are unfamiliar with the sales tax rates in the area. You are given the sales receipt shown at the left. Find the sales tax rates indicated by the receipt.

SOLUTION

For sales tax, the retail price is always the **base**.

$$\begin{array}{l} \text{City sales tax} \rightarrow \frac{1.36}{135.95} \approx 0.01 = 1\% \\ \text{Retail price} \rightarrow 135.95 \end{array}$$

The city has a sales tax rate of about 1%.

$$\begin{array}{l} \text{County sales tax} \rightarrow \frac{4.35}{135.95} \approx 0.032 = 3.2\% \\ \text{Retail price} \rightarrow 135.95 \end{array}$$

The county has a sales tax rate of about 3.2%.

$$\begin{array}{l} \text{State sales tax} \rightarrow \frac{9.25}{135.95} \approx 0.068 = 6.8\% \\ \text{Retail price} \rightarrow 135.95 \end{array}$$

The state has a sales tax rate of about 6.8%.



✓ Checkpoint

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- a. Suppose that Washington State loses the sales tax on 10,000 major appliances that are purchased in Oregon each year. The sales tax rate in Washington is 6.5%. Estimate the loss in tax revenue from these sales.
- b. Suppose that each person in the United States avoids paying \$500 in sales tax each year (by buying out of state, buying on the Internet, or buying at informal outlets such as garage sales). Estimate the total sales tax revenue lost each year.

Finding the Excise Tax on an Item

There are many types of **excise taxes**. The most common excise tax applies to a specific class of goods, typically alcohol, gasoline, cigarettes, and gambling. Rather than being a certain percent, excise taxes are often a fixed dollar amount. For instance, a state might charge \$1 per gallon of gasoline or \$2 per pack of cigarettes. The tax is reported to the consumer as part of the retail price. For instance, when gasoline is reported as \$2.94 per gallon, this price includes any state or federal excise tax on the gasoline.



The first federal excise tax in the United States was levied in 1791 on distilled whiskey. The tax was unpopular with farmers on the western frontier who got their corn to market by distilling it into whiskey. The unpopularity of the tax grew into a revolt known as the Whiskey Rebellion.

EXAMPLE 3 Estimating Excise Tax

Estimate the amount of excise tax generated by cigarette sales each year in the United States.

SOLUTION

This is not an easy question, but you can still come up with a reasonable estimate. Assume the following.

- There are about 230 million people who are 18 years old or older.
- Of these, about 20% smoke cigarettes.
- A cigarette smoker smokes one pack per day.
- State and federal excise taxes total about \$2.50 per pack.

With these assumptions, you can obtain the following estimate.

$$(230,000,000)(0.20)\left(\frac{1 \text{ pack}}{\text{day}}\right)\left(\frac{\$2.50}{\text{pack}}\right)\left(\frac{365 \text{ days}}{\text{year}}\right) = \frac{\$41,975,000,000}{\text{year}}$$

So, a reasonable estimate for the revenue generated by state and federal excise taxes on cigarettes is about \$40 billion.

✓ Checkpoint

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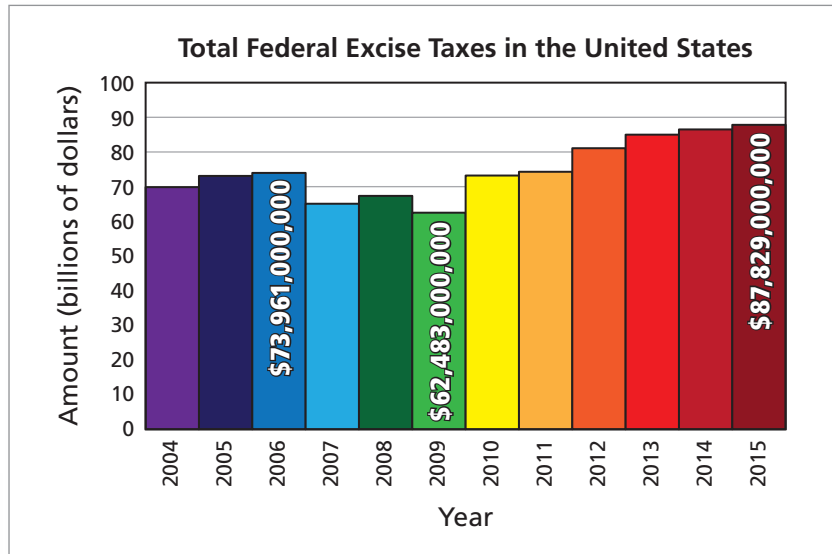
The United States uses about 140 billion gallons of gasoline in a year. Estimate the amount of excise tax generated by gasoline sales each year in the United States.



Excise taxes on tobacco, alcohol, and gambling are sometimes called “sin taxes.”

EXAMPLE 4 Finding Changes in Excise Taxes

The graph shows the total excise tax revenue (actual and projected) that the U.S. federal government raises.



The Telephone Excise Tax Refund was a onetime payment available on the 2006 federal income tax return. It refunded excise taxes collected on long-distance telephone services billed between February 28, 2003, and August 1, 2006. Most phone customers, including most cell phone users, qualified for the refund.

- How much did the revenue decline from 2006 to 2009?
- Why did the revenue decline during those years?

SOLUTION

- From 2006 to 2009, the revenue declined by

$$73,961,000,000 - 62,483,000,000 = \$11,478,000,000.$$

This is a decline of about 15.5%.

- One cause for the decline in revenue was an economic recession that began in 2007. There was, however, a less obvious cause. In 1898, the U.S. federal government implemented the first telephone excise tax in an effort to help fund the Spanish-American war. In 2005, a federal court declared it illegal to tax long-distance telephone services, a decision that cost the federal government billions of dollars in revenue.

Checkpoint

Help at Math.andYOU.com

Use the graph above to answer the questions.

- How much is the revenue projected to increase from 2009 to 2015?
- What is the percent increase, using 2009 as the base year?
- Suppose you are asked to devise a plan to raise the total excise tax revenue to \$250 billion by 2015. Describe such a plan and explain why you think it could raise the necessary tax revenue.

Value-Added Tax

A **value-added tax** (VAT) is a consumption tax. It is a tax on the market value that is added to a product or material at each stage of its manufacture or distribution. The tax is passed on to the consumer because it increases the ultimate retail price of the product. Value-added taxes differ from sales taxes because sales taxes are applied only at the point of purchase.

As of the writing of this text, the United States does not have a value-added tax. It has been considered by Congress but has always been rejected. Several other countries, including Canada, do have value-added taxes.

Proponents of value-added taxes argue that they inhibit consumer spending less than sales taxes, because they are hidden in the final retail price of the product.

EXAMPLE 5 Comparing Sales Tax and Value-Added Tax

During the production of a sheepskin coat, value is added by the following.

- A farmer: \$10
- A leather tanning company: \$25
- A coat making company: \$150
- A retail clothing company: \$210

Compare the effect of (a) a sales tax of 10% and (b) a value-added tax of 10%.

SOLUTION

- a. A sales tax of 10% is straightforward and easy to calculate. No consumption taxes are paid until the coat is sold to the consumer.

$$\text{Sales tax} = (0.10)(10 + 25 + 150 + 210) = \$39.50$$

The consumer pays \$434.50, of which \$39.50 is sales tax.

- b. Following the trail of a value-added tax is more complicated. A spreadsheet helps keep track of the value-added tax at each stage.



The rules for collecting and paying a value-added tax vary. However, as you can see in Example 5, value-added taxes involve more paperwork.



	A	B	C	D
1	Current Value	Value Added	10% Value-Added Tax	New Value
2	\$0.00	\$10.00	\$1.00	\$11.00
3	\$11.00	\$25.00	\$2.50	\$38.50
4	\$38.50	\$150.00	\$15.00	\$203.50
5	\$203.50	\$210.00	\$21.00	\$434.50
6	Total	\$395.00	\$39.50	
7				

The total tax is the same: \$39.50.

✓ Checkpoint

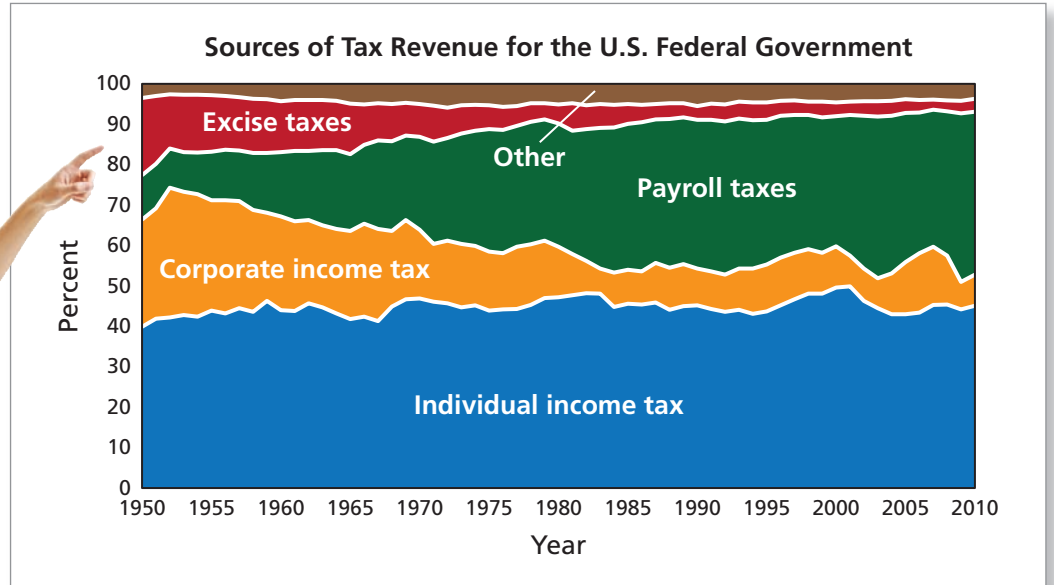
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In the example above, compare the effects of (a) a sales tax of 8% and (b) a value-added tax of 8%.

EXAMPLE 6 Comparing Tax Revenue Sources

The stacked area graph shows the sources of tax revenue for the U.S. federal government.

Since 1950, excise taxes have become a smaller and smaller percent of the total tax revenue for the federal government.



Describe the changes in the types of taxes from 1950 to 2010.

SOLUTION

Individual income tax This source of tax revenue has held roughly constant during the 60-year period. It makes up about 45% of the total tax revenue.

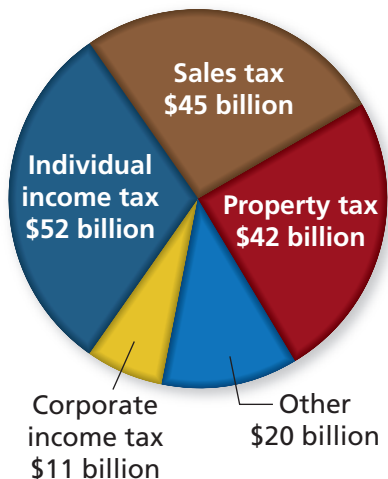
Corporate income tax This source of tax revenue declined from about 30% in 1950 to about 10% in 1985. From that time on, it has held fairly steady at about 10%.

Payroll taxes This source of tax revenue increased from about 10% in 1950 to about 35% in 1985. From that time on, it has held fairly steady at about 35%.

Excise taxes This source of tax revenue decreased from about 20% in 1950 to only about 3% in 2010.

Other This source of tax revenue has held roughly constant during the 60-year period. It makes up about 5% of the total tax revenue.

State and Local Taxes in California



Checkpoint

Help at Math.andYOU.com

The circle graph shows the total tax revenue for state and local governments in California during a recent year.

- Compare the percent of California’s tax revenue raised from individual income taxes with the percent of federal tax revenue raised from individual income taxes.
- Compare the percent of California’s tax revenue raised from corporate income taxes with the percent of federal tax revenue raised from corporate income taxes.

2.3 Exercises

Oil Painting In Exercises 1–6, use the table and the oil painting shown. (See Examples 1 and 2.)

U.S. State Sales Tax Rates (%) in 2010									
Alabama	4.0	Hawaii	nil	Massachusetts	6.25	New Mexico	5.0	South Dakota	4.0
Alaska	nil	Idaho	6.0	Michigan	6.0	New York	4.0	Tennessee	7.0
Arizona	5.6	Illinois	6.25	Minnesota	6.875	North Carolina	5.75	Texas	6.25
Arkansas	6.0	Indiana	7.0	Mississippi	7.0	North Dakota	5.0	Utah	4.7
California	8.25	Iowa	6.0	Missouri	4.225	Ohio	5.5	Vermont	6.0
Colorado	2.9	Kansas	5.3	Montana	nil	Oklahoma	4.5	Virginia	5.0
Connecticut	6.0	Kentucky	6.0	Nebraska	5.5	Oregon	nil	Washington	6.5
Delaware	nil	Louisiana	4.0	Nevada	6.85	Pennsylvania	6.0	West Virginia	6.0
Florida	6.0	Maine	5.0	New Hampshire	nil	Rhode Island	7.0	Wisconsin	5.0
Georgia	4.0	Maryland	6.0	New Jersey	7.0	South Carolina	6.0	Wyoming	4.0

- You buy the oil painting in an art gallery in California. What is the state sales tax?
- You buy the oil painting in an art gallery in Missouri. What is the state sales tax?
- You buy the oil painting in Washington, D.C. The sales receipt shows that you paid \$75.54 in sales tax. What is the sales tax rate in Washington, D.C.?
- You buy the oil painting in New York. The sales receipt shows that you paid \$88.13 in county sales tax. What is the county sales tax rate?
- You can buy the oil painting in Mississippi or drive 20 miles to an art gallery in Alabama to make the purchase. Which option would you choose? Explain your reasoning.
- Under what conditions is it beneficial to a consumer to drive to another state to make a purchase? Give specific examples.



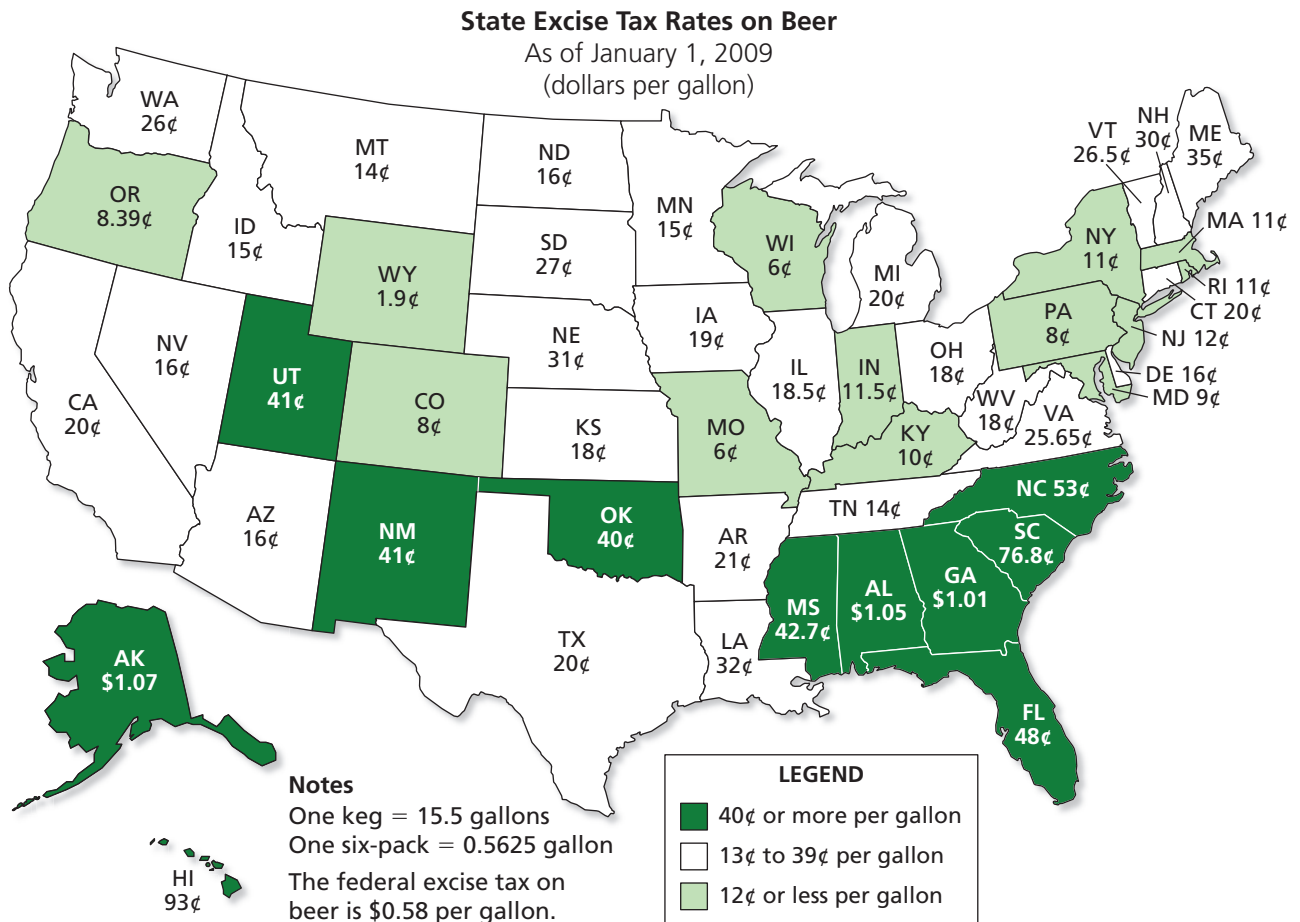
Oil painting: \$1259.00

T-shirt	\$25.00
SUBTOTAL:	\$25.00
Sales tax city:	\$ 0.30
Sales tax state:	\$ 1.50
TOTAL:	\$26.80

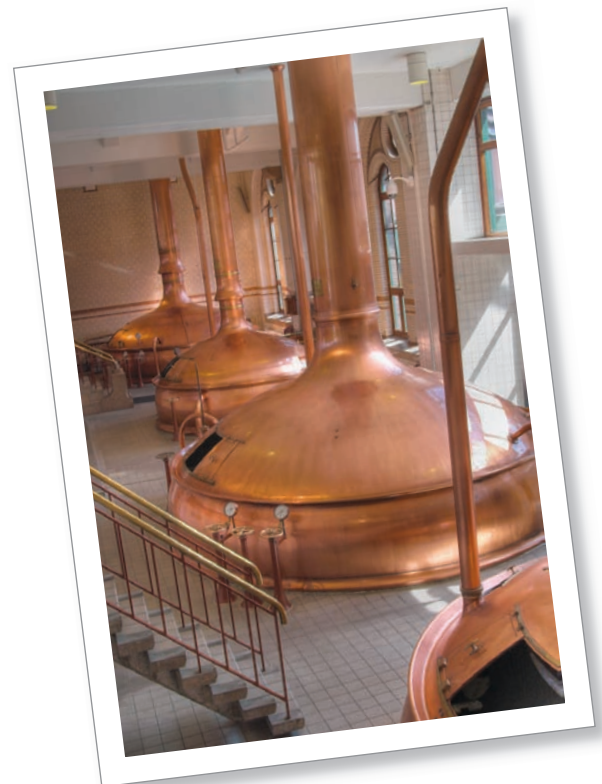
-- THANK YOU --

- T-shirt** You are given the sales receipt shown after buying a T-shirt. Find the sales tax rates indicated by the receipt. (See Example 2.)
- Internet Sales** Some people avoid paying sales tax by purchasing items out of state. This is a big problem for many states. What would you do as a state legislator to fix the problem?

Beer Tax In Exercises 9–13, use the information in the map. (See Examples 3 and 4.)



9. In which state is the excise tax rate on beer the least?
the greatest?
10. What is the state excise tax on a keg of beer in Georgia?
in Idaho?
11. What is the state excise tax on a 6-pack of beer in Alaska?
12. A customer in Florida buys a 12-pack of beer. How
much does the customer pay in federal and state
excise taxes?
13. A bar manager in South Carolina purchases a keg of beer
for \$110.00. What percent of the cost is allocated to federal
and state excise taxes?
14. **Excise Tax vs. Sales Tax** How is an excise tax
different from a sales tax? Explain your reasoning.





Car Manufacturing In Exercises 15–17, use the information below. (See Example 5.)

During the production of a car, value is added by the following.

- Raw materials manufacturers: \$7000
- A car manufacturer: \$11,000
- A car dealer: \$5000



15. The sales tax rate is 6.0%. What is the sales tax on the car?
16. Use the spreadsheet to find the value-added tax of 6% at each stage. Compare the value-added tax of 6% with a sales tax of 6%.

	A	B	C	D	E
1		Current Value	Value Added	6% Value-Added Tax	New Value
2	Raw materials	\$0.00	\$7,000.00		\$7,420.00
3	Manufacturer	\$7,420.00			
4	Dealer				
5		Total			
6					

17. Using the value-added tax approach, what is the retail price of the car?



Truck Manufacturing In Exercises 18–20, use the information below. (See Example 5.)

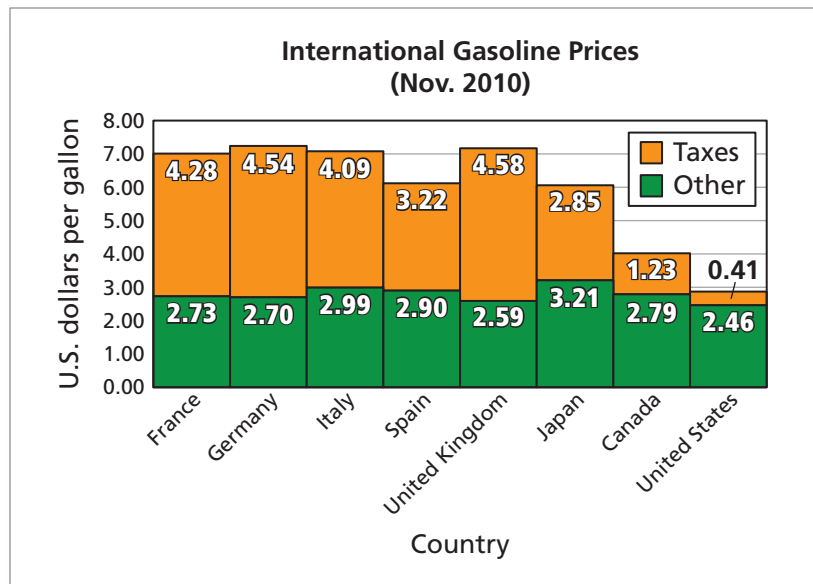
During the production of a truck, value is added by the following.

- Raw materials manufacturers: \$11,000
- A truck manufacturer: \$16,000
- A truck dealer: \$7000

18. The sales tax rate is 8.25%. What is the sales tax on the truck?
19. Use a spreadsheet to find the value-added tax of 8.25% at each stage. Compare the value-added tax of 8.25% to a sales tax of 8.25%.
20. Using the value-added tax approach, what is the retail price of the truck?


Gasoline The graph shows the prices of gasoline in eight countries. In Exercises 21 and 22, use the graph. (See Example 6.)

21. Use percent to compare the taxes collected per gallon of gasoline in the United States with those collected in Germany.
22. Describe any patterns you see in the graph.



▶ Extending Concepts


Sunglasses In Exercises 23 and 24, use the sales receipt.



ITEM		
Hotshot sunglasses	\$120.00	
<hr/>		
SUBTOTAL	\$120.00	
Shipping	\$0.00	
Sales tax	\$8.40	
<hr/>		
TOTAL	128.40	
-THANK YOU-		

23. While on vacation in Florida, you buy a pair of sunglasses for the beach. The state sales tax rate in Florida is 6%. What is the local sales tax rate?
24. Your friend loves the sunglasses you bought while on vacation in Florida. She decides to buy the same pair of sunglasses from the Ray's Beachwear website. What might be the total cost for her order? Explain.

25. **Exempt** Some items are not subject to sales tax in certain tax jurisdictions. The rules for exempt status can vary greatly from state to state. The state sales tax rate applied on the sales receipt shown at the right is 6%. Which item is exempt from sales tax?
26. **Other Excise Taxes** One of the main purposes of an excise tax is to discourage certain behavior. Name one item or service (not listed in this text) that you think should have an excise tax. Explain how you would define the tax structure, including the rate.



6 in. ham & cheese sub	\$4.50
Bottle of cough syrup	\$7.99
Pack of pens	\$1.99
<hr/>	
SUBTOTAL:	\$14.48
STATE SALES TAX:	\$0.39
<hr/>	
TOTAL:	\$14.87
THANK YOU	



27. **Dining Room Table Set** The value-added tax spreadsheet shows each stage of the production of a dining room table set. Complete the spreadsheet. What is the tax rate?

	A	B	C	D	E
1		Current Value	Value Added	Value-Added Tax	New Value
2	Raw materials	\$0.00	\$1,200.00		
3	Manufacturer	\$1,290.00	\$2,300.00		
4	Finisher	\$3,762.50	\$500.00		
5	Retailer	\$4,300.00	\$1,400.00		
6		Total	\$5,400.00		

28. **National Sales Tax** There has been some discussion about whether the United States should implement a national sales tax in addition to existing state and local sales taxes. Suppose you are entering a debate about a national sales tax. Use the Internet to research the topic. Would you be *in favor of* or *opposed to* a national sales tax? Explain your reasoning.



2.4 Budgeting

- ▶ Create and balance a monthly budget.
- ▶ Write checks and balance a checkbook.
- ▶ Analyze a budget.

Creating and Balancing a Monthly Budget

A personal monthly **budget** is a plan that includes your income and your expenses. Here are some tips for creating a meaningful budget.

- If you have bills that are due quarterly, semiannually, or annually, include a monthly average in your monthly budget.
- Include a savings goal in your budget. Saving for a goal is not only smart financially, it is smart emotionally.
- Leave some leeway in your budget for unexpected expenses.
- Keep your budget in a paper or an electronic journal. Save your receipts. Do not fall behind on entering expenses in your budget.



EXAMPLE 1 Comparing Budgeted and Actual Expenses

Find the difference between the actual amounts and the budgeted amounts.

CATEGORY	MONTHLY ACTUAL AMOUNT	−	MONTHLY BUDGETED AMOUNT	=	DIFFERENCE
Food					
Groceries	\$287.60		\$300.00		
Eating out, lunches, snacks	\$234.86		\$200.00		
Health and Medical					
Insurance (medical, dental, vision)	\$165.00		\$165.00		

SOLUTION

For each row, subtract the budgeted amount from the actual amount.

$$\begin{aligned} \text{Groceries:} & \quad 287.60 - 300.00 = -\$12.40 \\ \text{Eating Out:} & \quad 234.86 - 200.00 = \$34.86 \\ \text{Insurance:} & \quad 165.00 - 165.00 = \$0.00 \end{aligned}$$

✓ Checkpoint

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Find the difference between the actual amounts and the budgeted amounts.

CATEGORY	MONTHLY ACTUAL AMOUNT	−	MONTHLY BUDGETED AMOUNT	=	DIFFERENCE
Utilities					
Electricity	\$121.46		\$125.00		
Water and sewer	\$62.30		\$58.00		

Study Tip


When the actual amount is less than the budgeted amount, the difference is a negative number. In accounting, negative numbers are often indicated by parentheses and shown in red. For instance, $-\$12.40$ is written as **(\$12.40)**.

EXAMPLE 2 Balancing a Monthly Budget

Use the monthly budget form to find your total actual expenses. Compare your total actual expenses with your total budgeted expenses.

Study Tip

A negative number means you are under budget. So, it is “good” to have negative numbers for expense differences, but “bad” to have negative numbers for income differences.

 CATEGORY	MONTHLY ACTUAL AMOUNT	−	MONTHLY BUDGETED AMOUNT	=	DIFFERENCE
INCOME	\$5500.00		\$5500.00		\$0.00
Income Taxes Withheld					
Federal income tax	\$1100.00		\$1100.00		\$0.00
State and local income tax	\$225.00		\$225.00		\$0.00
Social Security/Medicare tax	\$412.50		\$412.50		\$0.00
Spendable Income	\$3762.50		\$3762.50		\$0.00
EXPENSES					
Home					
Mortgage or rent	\$455.00		\$455.00		\$0.00
Homeowners/renters insurance	\$95.00		\$95.00		\$0.00
Property taxes	\$234.00		\$234.00		\$0.00
Utilities					
Electricity	\$121.46		\$125.00		−\$3.54
Water and sewer	\$62.30		\$58.00		\$4.30
Natural gas	\$158.16		\$200.00		−\$41.84
Telephone (landline, cell)	\$138.92		\$125.00		\$13.92
Food					
Groceries	\$287.60		\$300.00		−\$12.40
Eating out, lunches, snacks	\$234.86		\$200.00		\$34.86
Health and Medical					
Insurance (medical, dental, vision)	\$165.00		\$165.00		\$0.00
Medical expenses, co-pays	\$0.00		\$200.00		−\$200.00
Transportation					
Car payments	\$175.00		\$175.00		\$0.00
Gasoline/oil	\$48.23		\$60.00		−\$11.77
Auto repairs/maintenance/fees	\$0.00		\$50.00		−\$50.00
Auto insurance	\$125.00		\$125.00		\$0.00
Debt Payments	\$253.48		\$253.48		\$0.00
Entertainment/Recreation	\$124.50		\$150.00		−\$25.50
Clothing	\$0.00		\$50.00		−\$50.00
Investments and Savings	\$125.00		\$125.00		\$0.00
Miscellaneous	\$93.50		\$200.00		−\$106.50
Total Expenses			\$3345.48		

SOLUTION

The total of your actual expenses is \$2897.01. So, you spent $3345.48 - 2897.01 = \$448.47$ less than what you budgeted.

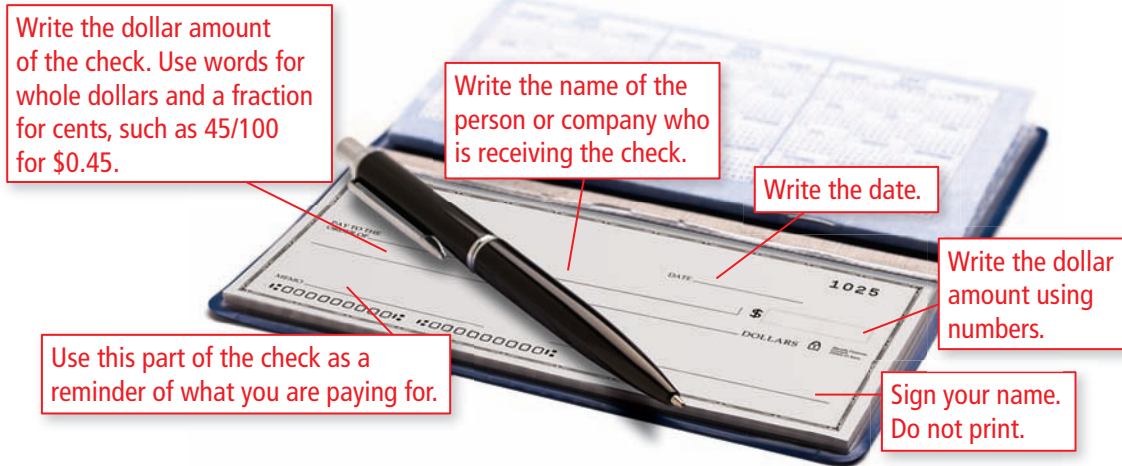
✓ Checkpoint

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Complete the “difference” column of the monthly budget form. Explain how you can use the total in this column to check the total in the “actual” column.

Writing Checks and Balancing a Checkbook

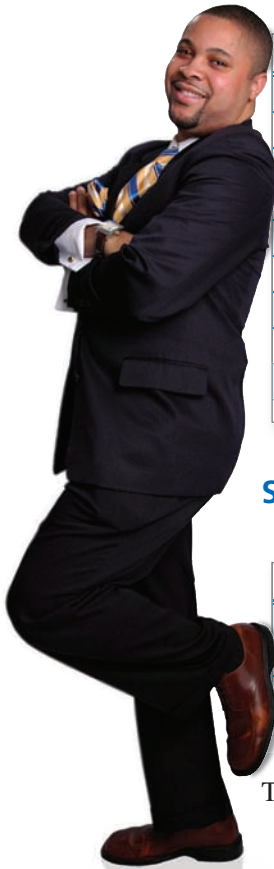
A **check** is a written order directing a bank to pay money. Checks are written on preprinted forms as shown.



Checkbooks come with a registry so that you can keep track of each transaction in your checking account.

EXAMPLE 3 Keeping a Checkbook Registry

Find the balance in your checking account as of 5/11/12.



Date	Check #	Transaction	Credit	Debit	Balance
		Balance Forward			100.00
5/11/12	996	Grocery Store		29.55	
5/11/12		Deposit Paycheck	482.75		
5/16/12		ATM Withdrawal		100.00	
5/17/12	997	Gym Membership		22.35	
5/17/12	998	Cell Phone Company		58.00	
5/17/12	999	Car Payment		82.66	
5/18/12		Deposit Paycheck	501.50		
5/18/12	1000	Birthday Gift		41.28	

SOLUTION

Date	Check #	Transaction	Credit	Debit	Balance
		Balance Forward			100.00
5/11/12	996	Grocery Store		29.55	70.45
5/11/12		Deposit Paycheck	482.75		553.20

$100.00 - 29.55 = \$70.45$
 $70.45 + 482.75 = \$553.20$

The balance as of 5/11/12 is \$553.20.

✓ Checkpoint

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Find the balance in your checking account as of 5/18/12.

A **bad check** is a check that is written for an amount that is greater than the balance in the checking account. A bad check is also called an *insufficient funds check* or a *bounced check*. Many banks charge penalties for writing bad checks. These penalties can be significant and can result in numerous problems.

In addition to bank and possible vendor penalties, writing a bad check is illegal. The penalty varies by state but can involve a fine and a prison term. Whether the state takes legal action usually depends on whether a person intentionally or accidentally writes a bad check.

EXAMPLE 4 Calculating Bad Check Penalties

You forget to record an ATM withdrawal of \$100 (5/20/12) in your checkbook registry. This causes three checks to “bounce.” Your bank charges \$50 for each bounced check. You think the balance in your account as of 5/23/12 is \$2.63. What is the actual balance after the bad check penalties are deducted?



Some common reasons overdrafts occur are errors in an account register, failure to enter ATM or debit card transactions in a register, and temporary holds on deposits.

Date	Check #	Transaction	Credit	Debit	Balance
		Balance Forward			332.85
5/20/12	406	Cell Phone Company		219.45	113.40
5/23/12	407	Pharmacy		23.56	89.84
5/23/12	408	Electric Company		48.67	41.17
5/23/12	409	Credit Card Payment		38.54	2.63

SOLUTION

After the penalties are deducted, your checkbook registry will look like this.

DATA

Date	Check #	Transaction	Credit	Debit	Balance
		Balance Forward			332.85
5/20/12		ATM Withdrawal		100.00	232.85
5/20/12	406	Cell Phone Company		219.45	13.40
5/23/12	407	Pharmacy		23.56	13.40
5/23/12		Insufficient Funds Penalty		50.00	-36.60
5/23/12	408	Electric Company		48.67	-36.60
5/23/12		Insufficient Funds Penalty		50.00	-86.60
5/23/12	409	Credit Card Payment		38.54	-86.60
5/23/12		Insufficient Funds Penalty		50.00	-136.60

Returned to vendor, not paid. —

Returned to vendor, not paid. —

Returned to vendor, not paid. —

The actual balance in your account is -\$136.60.

✓ Checkpoint

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Suppose that each vendor in Example 4 also charges \$25 for a bounced check. How much does forgetting to record your ATM withdrawal cost you?

Analyzing a Budget

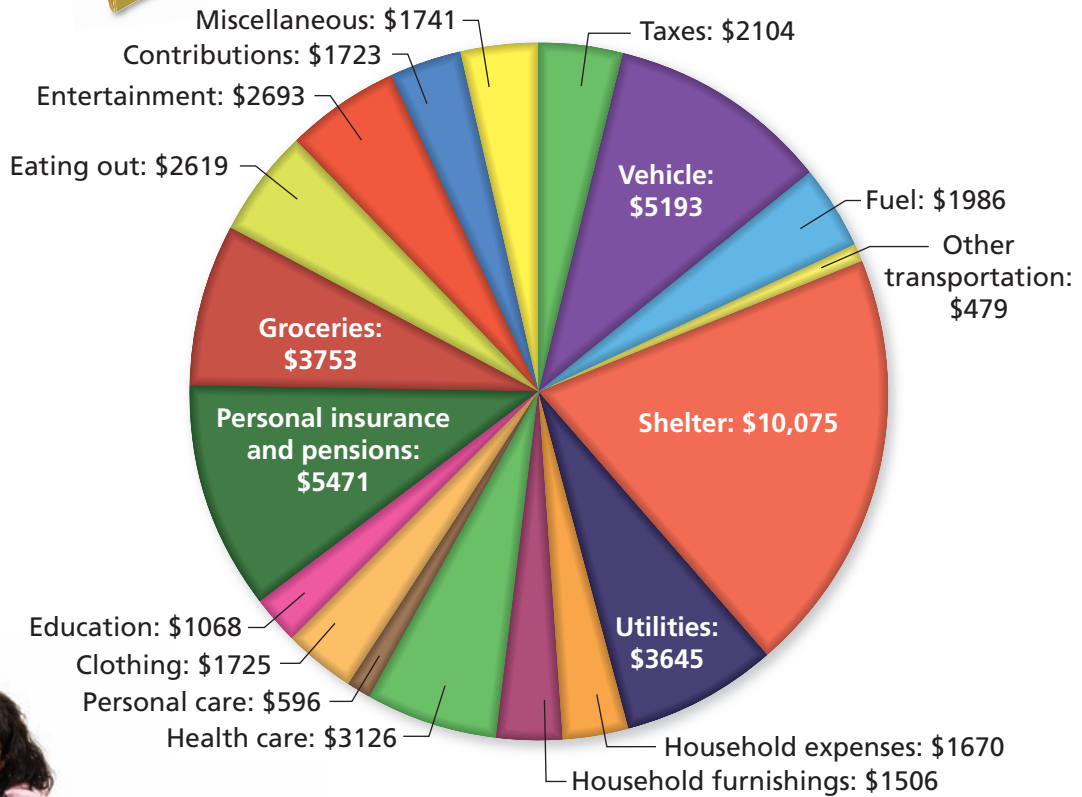
EXAMPLE 5 Analyzing Annual Expenses

The circle graph shows the average annual expenses for a U.S. household, according to the Department of Labor. What percent of the expenses are spent on taxes and shelter?

SOLUTION



Average Annual Expenses for a U.S. Household



The total of the expenses is \$51,173.

$$\frac{\text{Taxes} + \text{Shelter}}{\text{Total expenses}} = \frac{2104 + 10,075}{51,173}$$

$$\approx 0.238$$

$$= 23.8\%$$

The average household spends about 23.8% on taxes and shelter.

In the Department of Labor's survey, the average household had 2.5 members and 1.3 wage earners.



What percent of the expenses are spent on transportation? on food?

